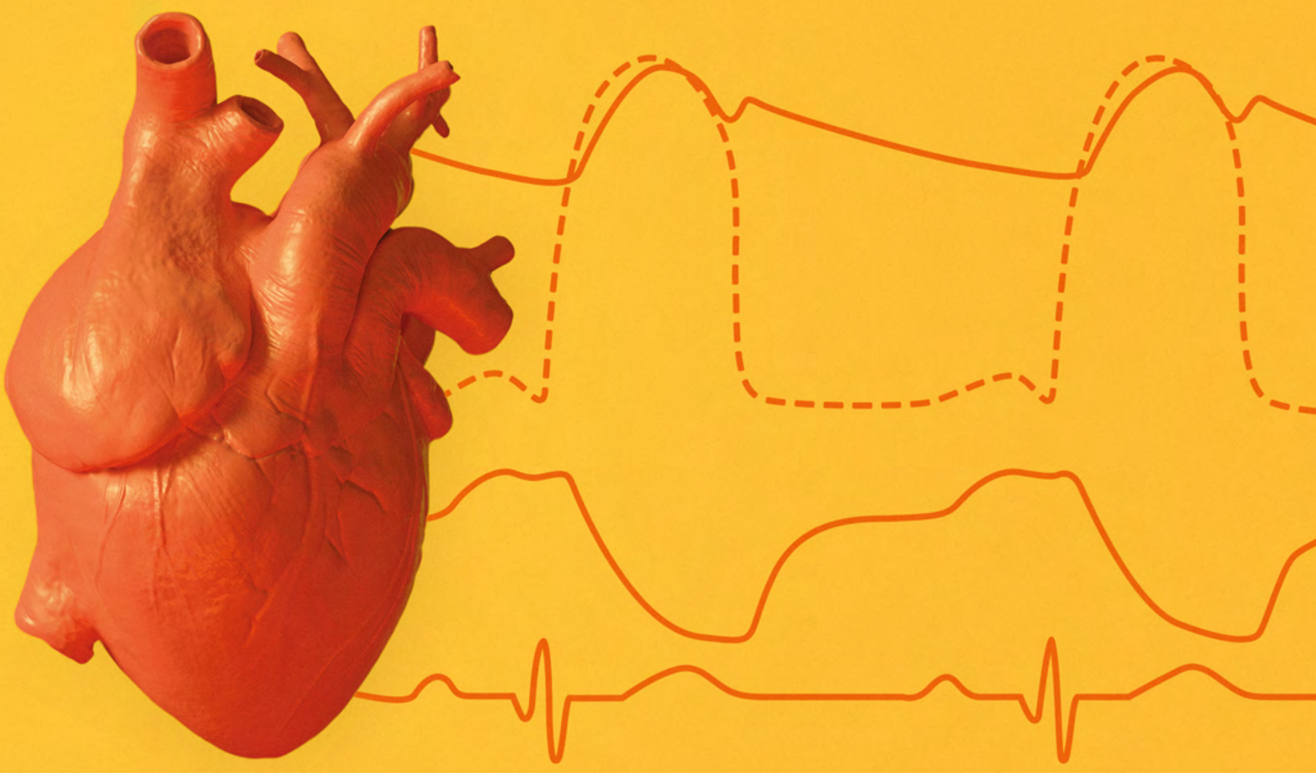
Alessandro Bacchetta   Domenico Scannicchio

# Introduction to Medical Physics



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# Introduction to Medical Physics

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# Foreword

In 2009, the University of Pavia launched the first Italian medical course taught in English, with the aim of creating an international school that would attract students from abroad as well as from Italy. We had the honor and the pleasure of teaching Medical Physics to the students of this course in the years between 2009 and 2014 (Prof. D. Scannicchio), and after 2015 (Prof. A. Bacchetta).

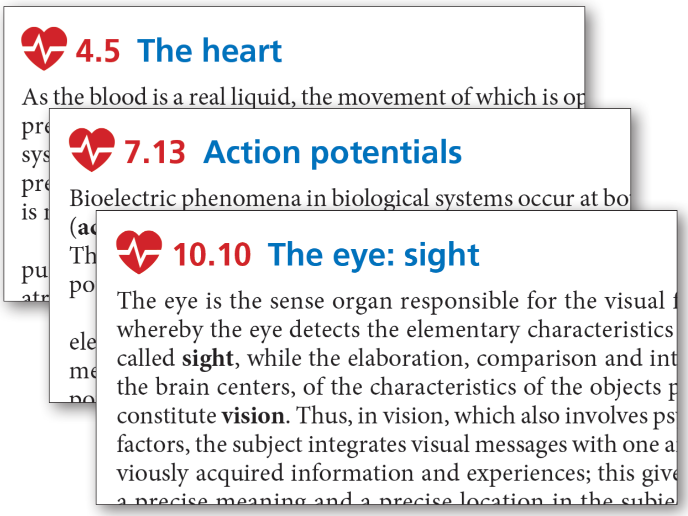
Many other international courses have been set up in Italy and in other European countries in recent years. However, we have noticed that there is a lack of suitable textbooks, i.e., texts that cover the topics usually dealt with in European courses with the appropriate level of sophistication. There are excellent textbooks in Italian, but they do not fit our context. There are also excellent textbooks in English, but they cover different topics and have a different approach (most of them are entirely algebra-based). So we decided to write our own book, taking advantage of the long-established teaching methods developed at our university, which was founded in 1361 and has one of the most prestigious medical faculties in Italy.

Writing this book in a language that is not our own has been a challenge. We apologize if our style does not sound very natural and elegant to English-speaking readers. However, our limitations have led us to focus more on content than on style. For this reason, we hope that the book will be a simple and concise resource for students who use English as a second language.

Similarly, we are physicists, not doctors. We are aware that our knowledge may be incomplete or outdated when discussing medically oriented concepts and applications. However, the aim of the book is not to be exhaustive and flawless, but rather to convey the idea that the concepts of Physics are essential in Medicine and that the hard science approach, whenever applicable, is the best means we have to solve problems, even in Medicine.

We have organized the book with the following goals in mind:

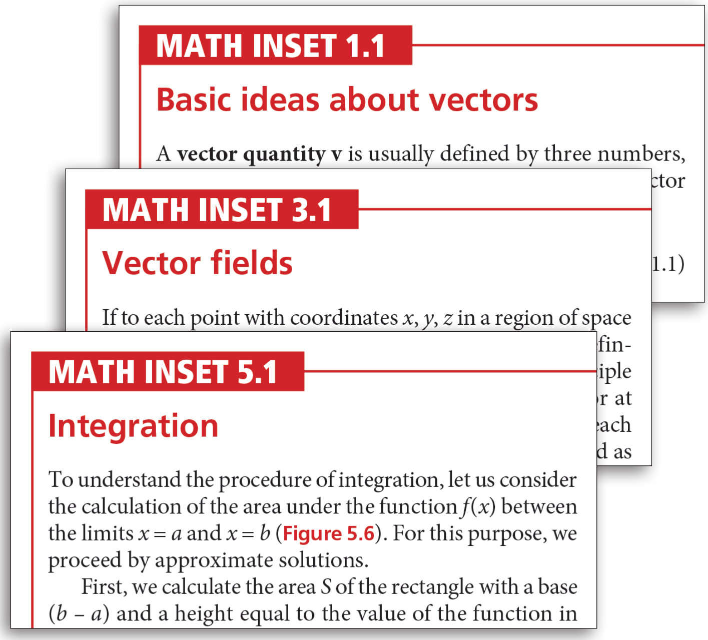
- to show how Physics can be used to explain some phenomena that occur in the human body, from the microscopic to the macroscopic level;
- to describe the physical principles that underlie modern medical instrumentation for diagnostic and therapeutic purposes;
- to provide a useful reference for students of other biomedical courses.




In each chapter, the sections devoted to biomedical applications are indicated by a heart-shaped icon; more than a third of the sections in the book are of this type.

As can be seen from the Table of Contents, we start with mechanics (Chapter 1) and its applications to the muscular and skeletal systems (Chapter 2). We devote Chapter 3 and 4 to fluid dynamics, as applied to the cardiovascular system. We then discuss gases and thermodynamics in relation to the respiratory system, human physiology and thermoregulation (Chapter 5). Chapter 6 is devoted to an analysis of the phenomenon of diffusion, in particular through membranes; although this topic is not usually included in introductory Physics courses, it is of great importance to Medicine. In Chapter 7, we review electrical phenomena and emphasize their role in cellular activity. Chapter 8 deals with the physics of wave phenomena, from sound waves to the perception of sound. Chapter 9 briefly reviews the concepts of magnetism that give rise to electromagnetic waves, which are now widely utilized in medical diagnostics and therapy. Chapter 10 considers the specific example of visible electromagnetic waves, i.e., light, and discusses optics and vision. Chapter 11 describes the structure of atoms and nuclei, in order to explain the nature of radiations, which are commonly exploited in diagnostic and therapeutic applications, and their interaction with biological systems. The last chapter (Chapter 12) deals with some important examples of biomedical instruments and recalls some of the notions discussed in the previous chapters.

As mathematics is the language of Physics, the book displays a certain level of mathematical sophistication. The students attending our courses are at the highest level: they deserve to be informed about the existence of certain mathematical tools and of how powerful and versatile they can be. These are described in several **MATH INSETS**. For instance, we have introduced derivatives and integrals, in simplified terms. However, the book can be read without any prior knowledge of Calculus.




In each chapter, we provide many examples and present problems, together with their solutions and numerical results. These not only show that Physics is useful in Medicine when it provides quantitative predictions, but also give the reader an indication of the orders of magnitude of the physical quantities encountered in biomedical problems.

We are aware that a standard, one-semester course in Physics for Medicine cannot cover all the subjects included in our textbook. Nevertheless, we hope that the book can be used as a flexible tool, both by teachers, who can select the topics and applications they consider most appropriate, and by students, who can also use the text as a reference manual for other courses, for specialist Master's Degrees, and for their future careers.

We would like to thank Prof. E. Giroletti, Prof. S. Bortolussi (Department of Physics, University of Pavia, Italy), Prof. R. Orecchia (Scientific Direction IEO Milan, Italy), Dr. F. Ceci (Division of Nuclear Medicine, IEO Milan, Italy), Prof. B.A. Jereczek, Dr. F. Cattani and their staff (Division of Radiotherapy and Division of Medical Physics, IEO Milan, Italy), Prof. F. Calliada (Echography, Mondino Hospital, Pavia, Italy), Prof. Ciro Esposito (Dialysis Service, ICS Maugeri, Pavia, Italy) and the CNAO and HIT Direction staff.

We would also like to thank dr. L. Fiorenza, who first believed in this editorial project, Dr. I. Nenci, Dr. A. Milani, all the Zanichelli staff and the English auditor Prof. B. Patrick.

## CHAPTER 8

# Waves and acoustics

## 8.1 Introduction: waves in nature and hearing

The wave motion and its propagation have considerable importance in the physical description of natural phenomena. In fact, there are numerous phenomena that show wave characteristics, such as sound waves, light waves, sea waves, seismic waves and, as we will see, all propagating periodic phenomena, such as the pulsatile motion of blood seen in Chapter 4.

Many aspects of this motion and its propagation can be generalized, which means that all wave phenomena can be described by means of similar physical-mathematical expressions. In the first part of this chapter, introductory notions of a general nature are provided; in the second part, these notions are applied to the description of sound phenomena and their applications in medicine. The study of mechanical vibrations that propagate in gases, liquids and solids (acoustics) is fundamental to understanding the mechanisms of reception, emission and transmission of these vibrations in humans and animals. Furthermore, as we will see, by using the propagation properties of mechanical waves, we can obtain non-invasive diagnostic information on the functionality of organs within biological systems, discussed in detail in Paragraph 12.7.

## 8.2 Description of waves

Let us consider a sea wave heading towards the coast. A float placed on the surface of the sea rises and falls without following the path of the wave. On observing the water molecules, we see that they oscillate around a position of equilibrium, without ever moving away from it. Waves transfer the disturbance (and in this way transfer energy) without transporting matter.

In general, a wave is a **disturbance of the state of equilibrium** of a system that **propagates through space**. Waves can be **periodic**, i.e. the function that describes the system repeats itself after a characteristic time (the **period** $T$). Sea waves are (almost) periodic. A sound wave associated with a specific note is a periodic wave. Light of a specific color is a form of periodic wave (electromagnetic wave, discussed in Chapters 9 and 10). In our body, the sphygmic wave (Paragraph 4.8) is an example of an (almost) periodic wave that propagates along the walls of the arteries. Many types of waves are **not periodic**: a tsunami, the sound of a melody, a short pulse of light, the action potential traveling along the axon (Paragraph 7.14)... Indeed, any wave with a beginning and an end is not periodic.

A sea wave is a propagating periodic wave, i.e., it is an oscillation that repeats itself after a characteristic time and moves through the medium. The same applies to vibrations propagating in a solid medium or along a string (**Figure 8.1**). Wave phenomena require the oscillation of a system around an equilibrium position (vibration), and are related to the existence of restoring forces, for instance elastic forces.

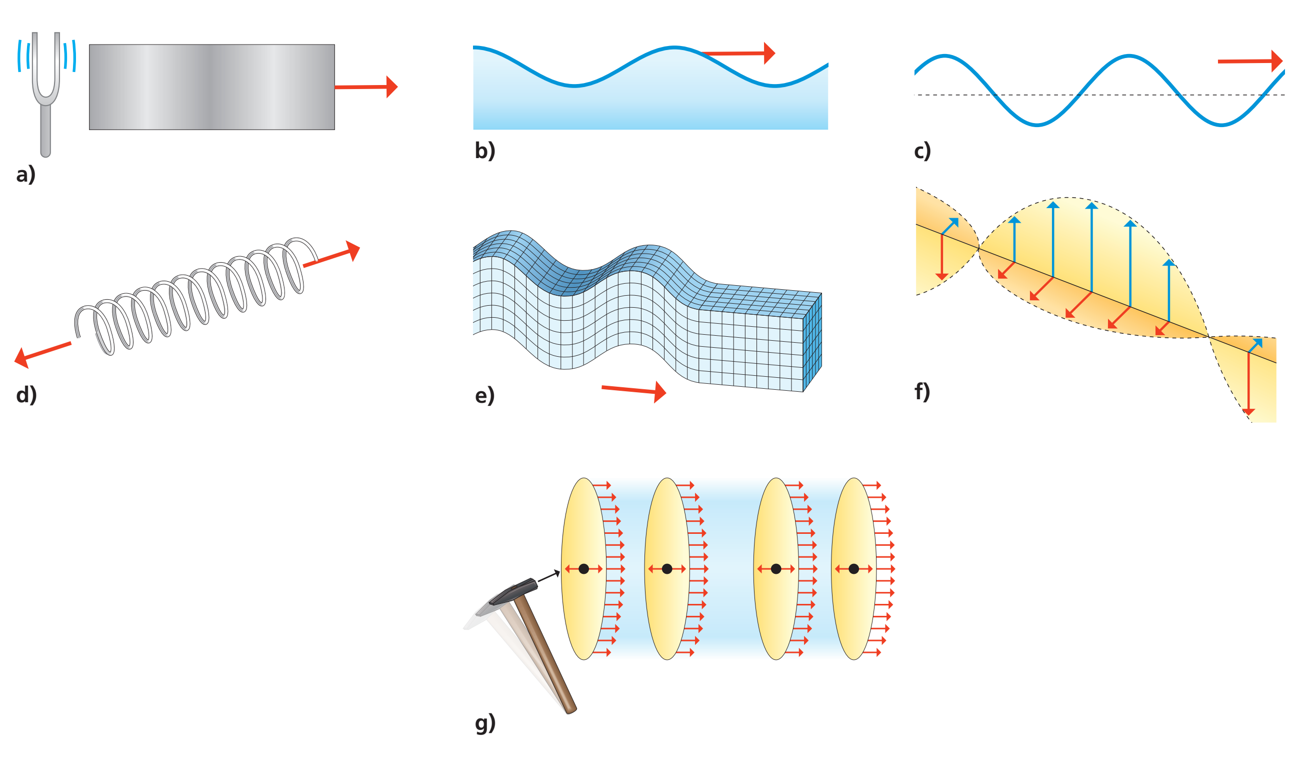



**Figure 8.1** Some wave phenomena: **a)** a sound wave (for example the vibration of a tuning fork sets in motion the molecules of the surrounding gas, which oscillate around their equilibrium position, creating volumes of compression and rarefaction); **b)** the surface wave propagating at the liquid-air interface (e.g., sea waves); **c)** a wave along a swinging rope; **d)** a wave through a spring; **e)** a type of seismic wave (S wave); **f)** a polarized (see Paragraph 8.7) electromagnetic wave, which is generated by the oscillation of electric and magnetic fields (described in detail in Chapter 9); **g)** elastic vibration propagating in a metal bar, where the atoms (black dots) oscillate back and forth around their equilibrium position.

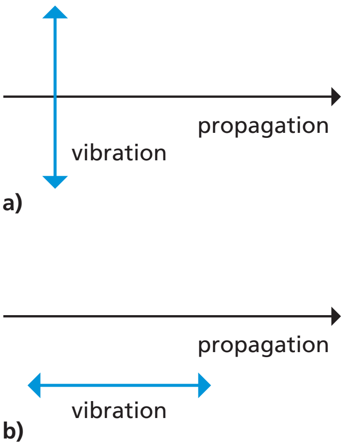



**Figure 8.2** Schematic representation of **a)** a transverse vibration and **b)** a longitudinal vibration.

See **MATH INSET 8.1**
Fourier (or harmonic) analysis
(p. 182)

Wave phenomena are generally divided into two categories. **Transverse waves** occur when the vibration is perpendicular to the direction of propagation, while **longitudinal waves** occur when the vibration is parallel to the direction of propagation (Figure 8.2). Electromagnetic waves, the wave propagating along a rope and the seismic *S* wave are transverse, while sound waves in air, waves in solids or along a spring are longitudinal.

In the following paragraphs, in order to simplify the initial description of wave phenomena, we will consider mechanical waves propagating in a single direction. However, these notions are applicable to any wave phenomenon.

## 8.3 Harmonic waves

As we saw in Chapter 1, a mass attached to a spring oscillates with a sinusoidal motion also called **harmonic motion**. Harmonic motion is the simplest type of a periodic phenomenon. A complex periodic motion can be described in terms of a superposition of harmonic motions. Understanding harmonic motion is thus extremely important.

A **harmonic wave** is the simplest type of periodic wave and is described by the function:

$$S(x,t) = A\sin(\omega t - kx + \varphi), \tag{8.1}$$

where $S(x,t)$ can be any physical quantity (e.g., water level, pressure difference, electric potential...), and is defined as a function of position and time. While a simple oscillation is a function of time only, a wave is a function of space and time, and represents the propagation of the oscillation through space.

**Figure 8.3a** shows the plot of a wave as a function of space at different instants of time, like a series of snapshots. The crests of the wave move to the right as time increases. The distance between two crests is the **wavelength** (2 m in this case). After a period (4 seconds in this case), the wave returns to the starting configuration. **Figure 8.3b** shows the same wave as a function of time at different positions, i.e., it shows what happens at certain fixed positions. The **period** is the time between the transit of two crests. Two points separated by a wavelength move in exactly the same way.

In Eq. (8.1), the coefficient $A$ is the **amplitude** of the wave, $\phi$ is the initial **phase angle**. The constant $\omega$, called **angular frequency** or **pulsation**, is related to the characteristic period $T$ of the vibration and is measured in **radians/s**. After one period, the function returns to its initial value, which means that the argument of the sine has to change by 2π radians, therefore

$$[\omega(t+T)+\phi]-(\omega t+\phi)=2\pi,$$

and

$$\omega T=2\pi, \quad \text{from which} \quad \omega=\frac{2\pi}{T}. \tag{8.2}$$

The **frequency** $f$ of a wave is defined as the number of oscillations performed in the unit of time and therefore, by definition, we have $f = 1/T$, and

$$\omega=\frac{2\pi}{T}=2\pi f. \tag{8.3}$$

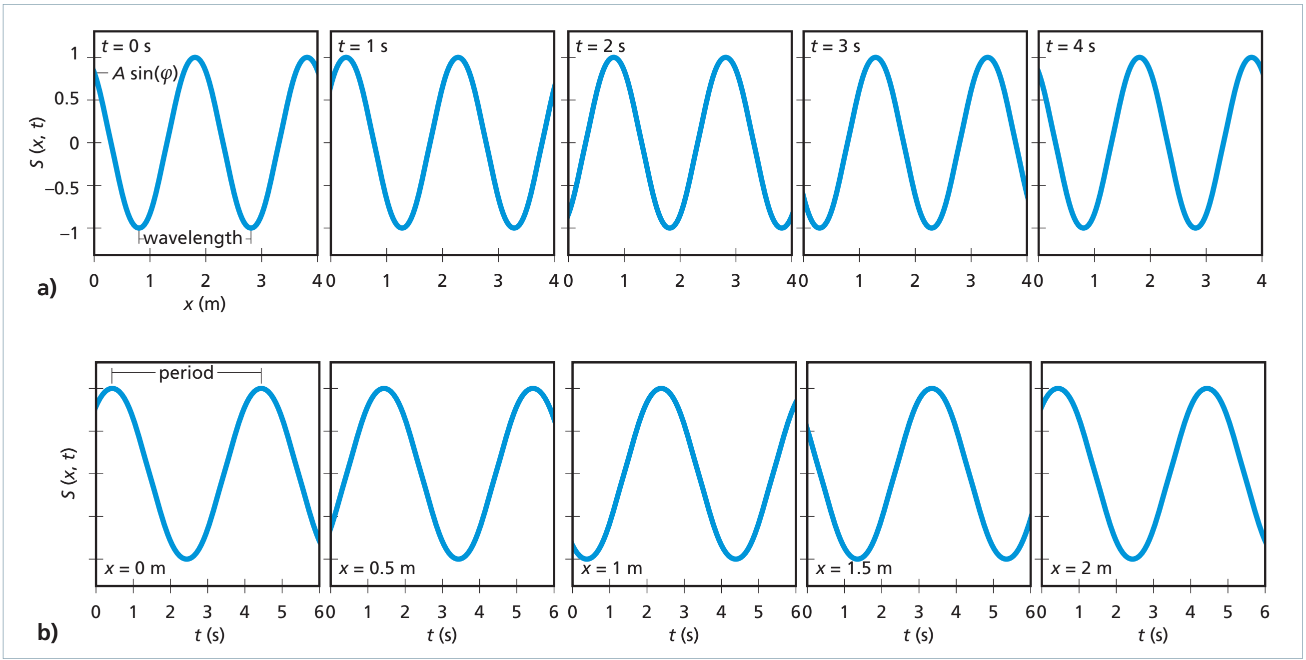



**Figure 8.3** Plots of a wave with a period of 4 s and a wavelength of 2 m. **a)** As a function of position at different instants of time. **b)** As a function of time at different positions.

The unit of measurement of frequency in the SI is the second$^{-1}$ called **hertz** (Hz).

See **MATH INSET 1.1**
Basic ideas about vectors (p. 4)

In Eq. (8.1), $k$ is the **wavenumber** (measured in radians per meter). In three dimensions, the product $kx$ is replaced by the scalar product $\mathbf{k}\cdot\mathbf{r}$ and $\mathbf{k}$ is called the **wavevector**. The **wavelength** of a wave is connected to the wavevector by the relation

$$\lambda = \frac{2\pi}{|k|}. \tag{8.4}$$

If the sign in front of $k$ is negative, as in Eq. (8.1), the wave is **right-moving**, i.e., it moves in the positive $x$ direction. If the sign is positive, the wave is **left-moving**, in the negative $x$ direction.

In conclusion the Eq. (8.1) can be written as (assuming $\phi = 0$ for semplicity)

$$S(x,t) = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right). \tag{8.5}$$

The **phase velocity** of the wave is the velocity of a wave crest. In a period, the crest moves by a full wavelength, therefore phase velocity can be obtained as

$$v = \lambda f. \tag{8.6}$$

See **MATH INSET 4.3**
Differential equations (p. 65)

In general, the function describing a wave can be determined by solving a partial differential equation called the **wave equation**:

$$\frac{\partial^2 S(x,t)}{\partial t^2} = v^2 \frac{\partial^2 S(x,t)}{\partial x^2}. \tag{8.7}$$

It is not difficult to check that harmonic (i.e., sinusoidal) waves, Eq. (8.1), are solutions of the wave equation, with $v = \lambda f$.

Harmonic waves are only one of the many possible solutions of the wave equation. However, any wave can be decomposed in terms of harmonic waves. In the **MATH INSET 8.1** we will show that any periodic (and even non-periodic) phenomenon can be represented in terms of sinusoidal functions such as Eq. (8.1).

## MATH INSET 8.1

### Fourier (or harmonic) analysis

A wave can be a very complex periodic or non-periodic function. An important mathematical theorem, demonstrated by J. Fourier, establishes that any **periodic function** $f(t)$ can be represented as a **Fourier's series**, i.e., the sum of a finite or infinite number of sine and cosine functions that have appropriate amplitudes and frequencies. Fourier's theorem is written as follows:

$$\begin{aligned} f(t) &= C_0 + S_1\sin(\omega t) + C_1\cos(\omega t) + \\ &+ S_2\sin(2\omega t) + C_2\cos(2\omega t) + \\ &+ S_3\sin(3\omega t) + C_2\cos(3\omega t) + \\ &= \sum_{i=0}^{i=\infty} S_i\sin(n\omega t) + C_i\cos(n\omega t). \end{aligned} \tag{M8.1}$$

▶

If the period of the function is $T$, then $f = 1/T$ is the **fundamental frequency** and $\omega = 2\pi f$. The first sine and cosine functions in the Fourier's series, with amplitudes $S_1$ and $C_1$, are called the **fundamental harmonics**. The frequencies of all other terms in the decomposition (**higher harmonics**) are all multiples of the fundamental frequency. The $S_i$ and $C_i$ coefficients are the amplitudes of the individual harmonics.

The following integral formulas allow us to obtain the coefficients $C_i$ and $S_i$:

$$C_o = \frac{1}{T}\int_0^T f(t)\,\mathrm{d}(t),$$
$$S_i = \frac{2}{T}\int_0^T f(t)\sin(i\omega t)\,\mathrm{d}(t), \qquad \text{(M8.2)}$$
$$C_i = \frac{2}{T}\int_0^T f(t)\cos(i\omega t)\,\mathrm{d}(t).$$

As an alternative to Eq. (M8.1) we can have the following decomposition:

$$f(t) = \sum_{i=0}^{i=\infty} D_i \sin(i\omega t + \phi_i). \qquad \text{(M8.3)}$$

where the Fourier coefficients are now the amplitudes $D_i$ and the phases $\phi_i$ that can be related to the amplitudes $S_i$ and $C_i$ of the previous decomposition by

$$D_i = \sqrt{S_i^2 + C_i^2} \qquad \text{and} \qquad \mathrm{tg}\,\phi_i = \frac{S_i}{C_i}. \qquad \text{(M8.4)}$$

**Figure M8.1a** shows the first terms of the Fourier series of a square wave (blue line). The function has an average value of zero, therefore $C_o = 0$. All the terms with cosines and all the even harmonics (with $2\omega$, $4\omega$… ) have coefficients equal to zero. The series can be written in the following way:

$$f(t) = \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(\omega t) + \ldots\right) \qquad \text{(M8.5)}$$

After summing the first three odd harmonics, the resulting curve in red starts to approach a square wave. **Figure M8.1b** shows the **amplitude spectrum** of the periodic function, i.e., the amplitudes of each individual harmonic. In this case, the higher the harmonic is, the smaller its amplitude is.

As another example, let us consider the behavior of the instantaneous velocity of the blood in the case of pulsatile flow (see Figure 3.16). Since it is a periodic function, we can perform its Fourier analysis. In this case, the period is 0.69 s. **Figure M8.2a** shows the original

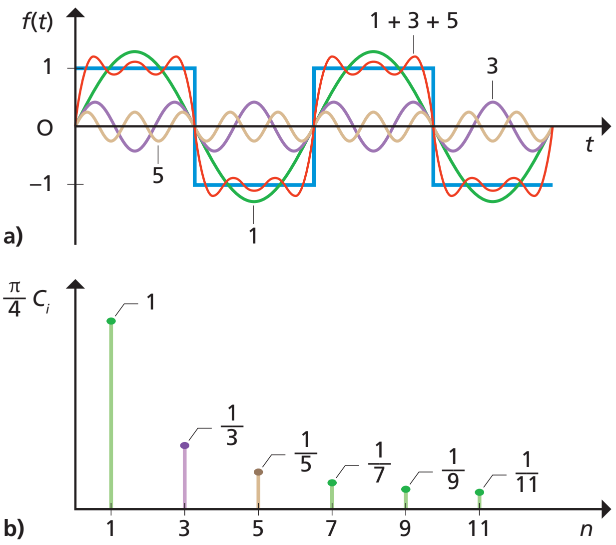



**Figure M8.1 a)** The first three odd harmonics of the Fourier analysis of a square wave. Adding many other harmonics gives a curve in red that is closer and closer to the square wave. **b)** Fourier spectrum of the square wave.

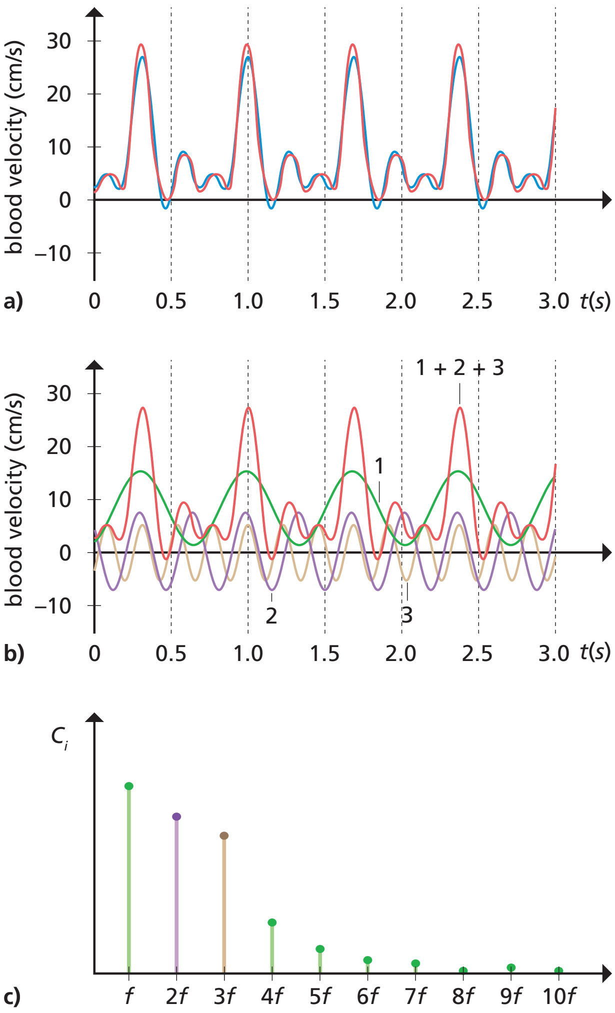



**Figure M8.2 a)** The complex behavior of blood velocity in an artery is described by a periodic function. **b)** This function can be Fourier analyzed and the figure shows the first three terms of the decomposition. **c)** Fourier spectrum of the previous function.

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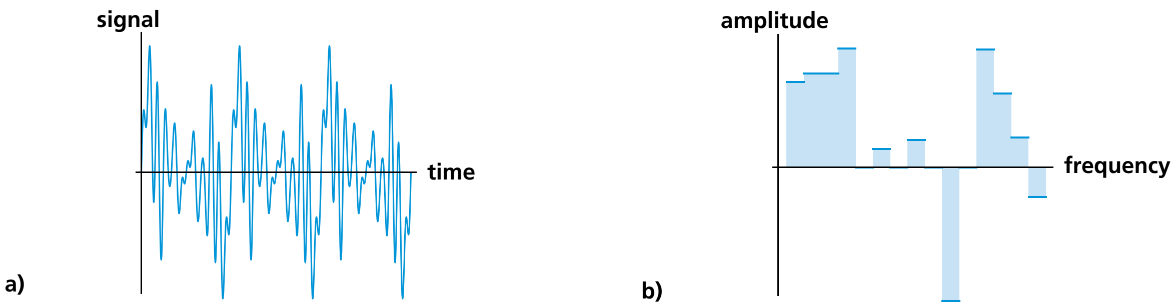
curve (blue) compared with the sum of the first three harmonics. The individual harmonics are shown in **Figure M8.2b** (notice that the first harmonic is displaced by the $C_0$ coefficient, i.e., the average velocity, which in this case is about 8.3 cm/s). A complex wave with its Fourier spectrum and Fourier power spectrum is shown in **Figure M8.3**. This theorem can also be extended to **non-periodic functions**; in this case, however, any frequency can contribute and the sum turns into an integral (Fourier's integral, see **MATH INSET 9.1**).



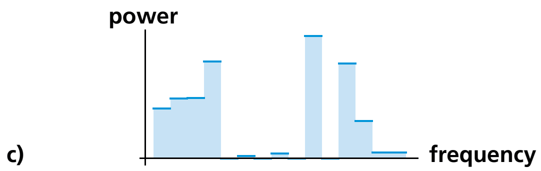




**Figure M8.3 a)** A complex periodic wave. **b)** Fourier amplitude spectrum (**a**) as a function of frequency. **c)** By squaring the amplitudes in **Fourier amplitude spectrum**, a Fourier power spectrum is obtained indicating which frequencies carry the most energy (Eq. 8.10). For much more complex vibrations the discrete amplitudes or their power are so piled close to each other as to originate a smooth function of the frequencies.

## 8.4 Wave propagation

In the propagation of mechanical vibrations, one point of the medium transmits the oscillation to its neighbors, i.e. to contiguous points. Owing to energy conservation, propagation of the wave involves the transmission of energy. The direction of energy flow is the **direction of propagation** of the wave, also called the **ray** of the wave (**Malus principle**). Wave **intensity** $I$ is defined as the energy transported in the unit of time and through the unit of surface perpendicular to the direction of propagation. In the SI, intensity is measured in **watts/m²**. In the case of curved surfaces and/or variable propagation directions, we can, as usual, take an infinitesimally small surface element and define at each point an energy flow intensity vector, or **energy current density vector**, which is similar to the concepts of heat flow intensity, solute current density, etc.

When studying elastic oscillations in Paragraph 1.10, we established that the potential energy of a particle of mass $m$ moving with harmonic motion is (from Eqs. (1.35) and (1.33))

$$U = \frac{1}{2}\omega^2 m S^2(t), \tag{8.8}$$

where $S(t)$ is given by Eq. (8.1) at $x = 0$, while the velocity of the particle is

$$v = \omega A \cos(\omega t + \phi). \tag{8.9}$$

The total energy of the particle is therefore

$$\begin{aligned} E_{\text{total}} = U + E_k &= \frac{1}{2}\omega^2 m S^2(t) + \frac{1}{2} m [\omega A \cos(\omega t + \phi)]^2 = \\ &= \frac{1}{2} A^2 \omega^2 m [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)] = \frac{1}{2} A^2 \omega^2 m, \end{aligned} \tag{8.10}$$

from which we see that the square of the amplitude $A$ of the oscillation is proportional to the total energy of the system. This result is very general and applies to waves as well: the **intensity of the wave is proportional to the square of its amplitude**.

As mentioned above, in the propagation of mechanical vibrations, a particle transmits the oscillation to its neighbors, i.e., to contiguous points. The **wavefront** is the set of all the points of the medium that are in the same state of vibration (i.e., they have the same phase), for example at the maximum value of the vibration amplitude.

In three dimensions, if the source is point-like, and energy is transmitted equally in all directions, the wavefronts are spherical (**Figure 8.4a**). The wave function depends only on time and the distance $r$ from the source:

$$S(r,t) = \frac{A_o}{r}\sin\left(\frac{2\pi t}{T} - \frac{2\pi r}{\lambda}\right), \tag{8.11}$$

where $A_o$ is the initial vibration amplitude. The energy distributed on the spherical surface of the wavefront of a wave with amplitude $A = A(r)$ is proportional to $A(r)^2r^2$. Since the energy has to be constant, we obtain $A(r) = A_o/r$.

According to **Huygens's principle**, any point on a wavefront becomes the source of a spherical wave, as shown in **Figure 8.5**.

A very important feature of wave propagation is **that waves with lower frequencies are less collimated**. This is due to the phenomenon of diffraction (discussed in Paragraph 10.3), which occurs when a wave meets an obstacle: the wave spreads around the obstacle (**Figure 8.6**). The amount of diffraction depends on the relative size of the obstacle and the wavelength of the waves. Waves

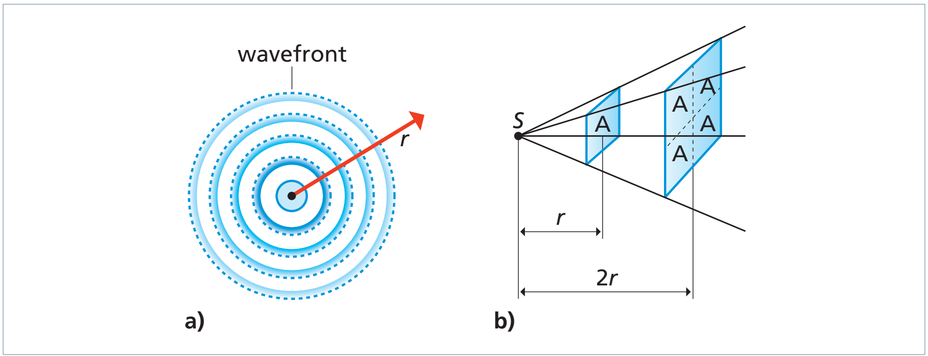



**Figure 8.4 a)** Schematic representation of a spherical wavefront: all points of the spherical wave surfaces vibrate with the same phase. **b)** The energy transported inside a fixed solid angle is conserved. Since the area of the wavefront increases as the square of the distance, the energy is distributed on the larger surface, and the intensity of the wave decreases accordingly.

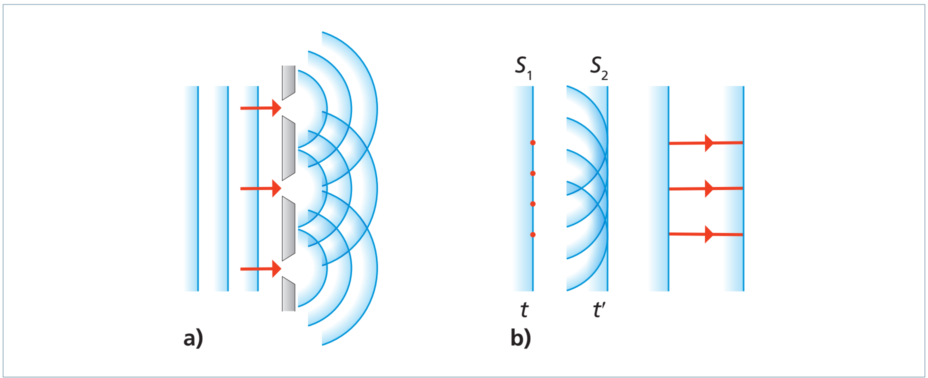



**Figure 8.5 a)** Semicircular wave surfaces arise at the openings. **b)** Generation of a plane wave surface: the envelope of the spherical wave surfaces forms a plane wave surface.

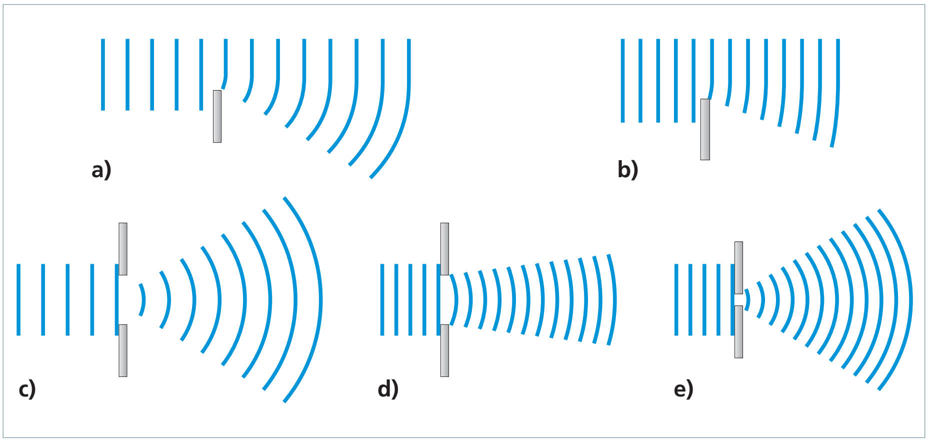



**Figure 8.6** When meeting an obstacle, the wave does not proceed in a straight direction, but spreads around the obstacle, a phenomenon called **diffraction**. Increasing the frequency the waves are more collimated (confront between **a)** and **b)** and between **c)** and **d)**. To obtain diffraction again from **d)** the slit must shrink as in **e)**.

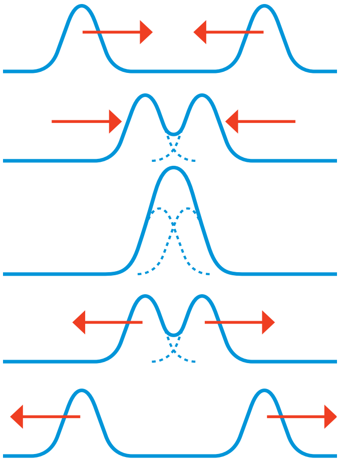


**Figure 8.7** Two waves (e.g., two vibrations of a string) traveling in opposite directions overlap: the resulting vibration at any point and instant of time is the sum of the vibrations due to each wave.

with longer wavelengths (lower frequencies) are more affected by diffraction and spread more. Waves with shorter wavelengths (higher frequencies) spread less (they are more collimated).

The **principle of superposition of waves** states that when two or more waves overlap, the net vibration at any point and at any instant of time is the sum of the individual vibrations due to each wave. If the vibrations occur in different directions, the net vibration is the vector sum of the individual vibrations (**Figure 8.7**).

These three principles are important in optics (Chapter 10) and in the interference phenomena (Paragraph 8.6).

## 8.5 Wave reflection and refraction

The phenomenon of **reflection** occurs when a wave reaches the interface between two different media. The laws of reflection state that

1) the incident ray, the reflected ray and the normal to the surface lie in the same plane, and
2) these rays form an angle of reflection $\hat{r}$ equal to the angle of incidence $\hat{i}$ (**Figure 8.8**).

Using Huygens's principle, the laws of reflection can be easily derived as shown in **Figure 8.9**.

The phenomenon of **refraction**, on the other hand, takes place when the wave is transmitted from one medium to another that has different physical properties. Also in this case, Huygens' principle can be used to obtain the refracted wavefront and therefore the laws of refraction (**Figure 8.10**). The laws of refraction (**Snell's laws**, **Figure 8.11**) state the following:

1) the incident ray, the refracted ray, and the normal to the surface separating the two media lie in the same plane;
2) owing to the different speeds of propagation in the two media, the following relationship is valid (**Figure 8.10**):

$$\frac{\sin \hat{i}}{\sin \hat{r}} = \frac{v_1}{v_2} = n_{12} = \frac{n_2}{n_1}, \tag{8.12}$$

where the constant $n_{12}$ is called the **relative refractive index** of the second medium with respect to the first, while $n_1$ and $n_2$ are the **absolute refractive indexes** of the two media.

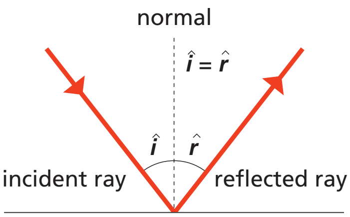



**Figure 8.8** Reflection of the propagation ray of a wave phenomenon from point P.

When a wave crosses the interface between a medium with a lower index of refraction (higher propagation speed) and one with a higher index of refraction (lower propagation speed), the refracted ray bends closer to the normal to the interface (**Figure 8.11**). Conversely, if the wave passes from a medium with a higher refractive index to one with a lower refractive index, when the angle of incidence exceeds an angle $i_o$ (**limit angle**), corresponding to an angle of refrac-

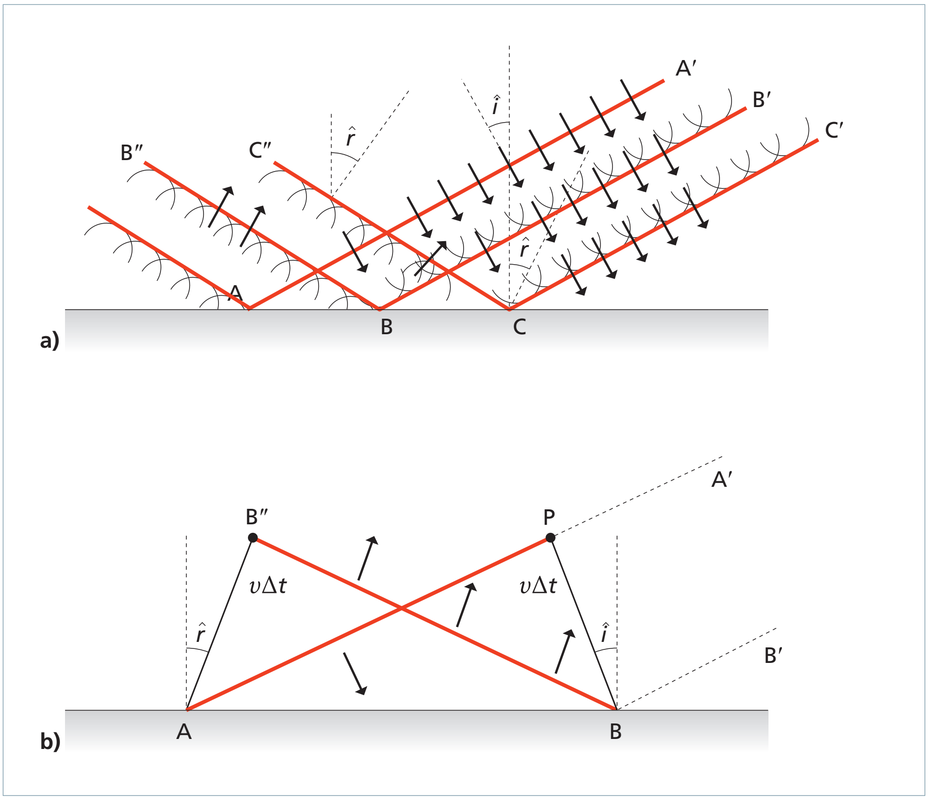



**Figure 8.9** **a)** A plane wave hits a flat surface. In chronological order, the wavefronts AA′, BB′ and CC′ touch the surface and points A, B and C become sources of spherical waves that propagate upwards, forming the reflected wavefronts AA″, BB″ etc. **b)** Application of Huygens' principle to the phenomenon of reflection. The wave surface AP initially meets the obstacle in A. After a time interval $\Delta t$, the elementary Huygens' wave coming from P at velocity $v$ arrives on the obstacle in B, while the one coming from A, at the same velocity reaches point B″. The triangles ABP and BAB″ are equal, the angles $\hat{i}$ and $\hat{r}$ are also equal ($\hat{i} = \hat{r}$).

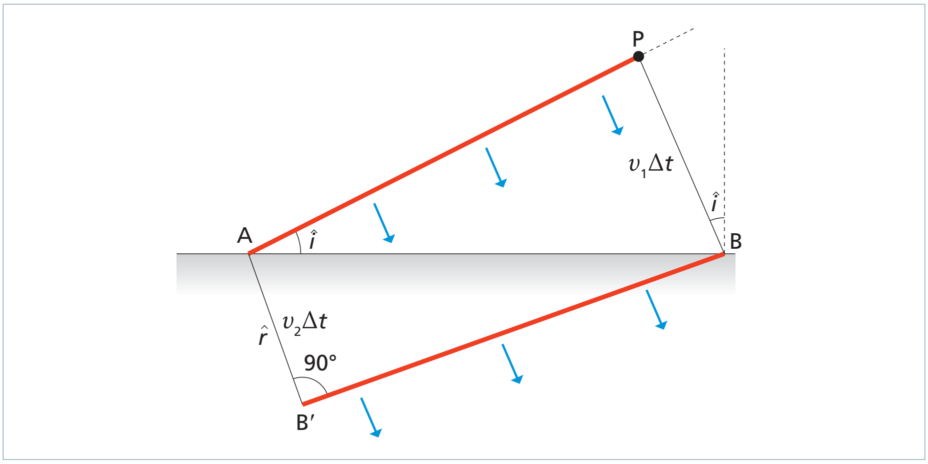



**Figure 8.10** A plane wave hits a plane that separates two media. Huygens' principle allows us to obtain the law of refraction. The incident wavefront AP initially strikes the plane at point A and, after a time interval $\Delta t$, the elementary Huygens' wave coming from P, at velocity $v_1$, reaches B. In the same interval $\Delta t$, the elementary wave emitted from A into the second medium reaches B′ and BB′ is the refracted wavefront. By applying simple trigonometric relations, we obtain: $AP \sin \hat{i} = v_1 \Delta t$ and $AP \sin \hat{r} = v_2 \Delta t$. On dividing the two equations member by member, we obtain the law of refraction (8.12).

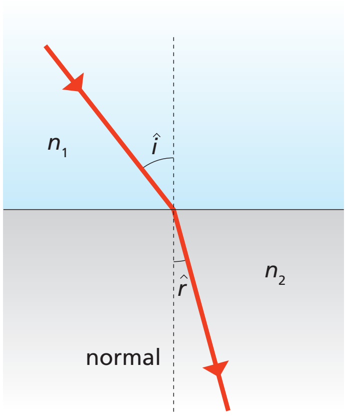



**Figure 8.11** Refraction of the propagation ray between two materials with different refractive index $n_1$ and $n_2$.

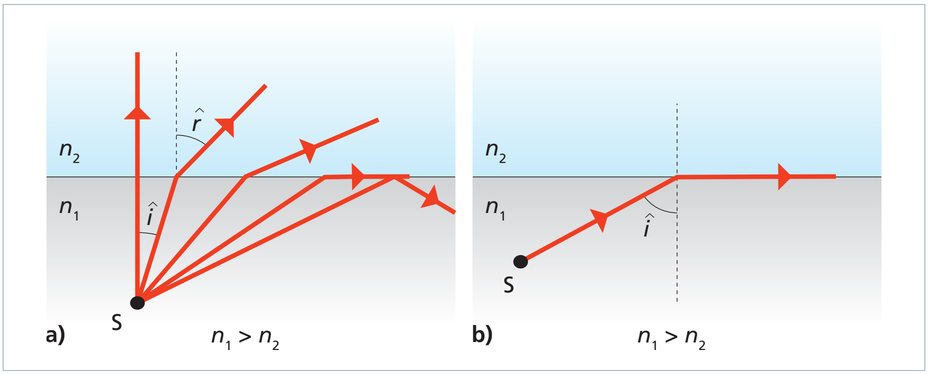



**Figure 8.12** Total reflection phenomenon: this occurs when the wave passes from a medium with a higher refractive index (lower propagation speed) to one with a lower refractive index (higher propagation speed).

tion of 90°, the intensity of the refracted ray vanishes and **total reflection** of the incident ray occurs (**Figure 8.12**).

Total reflection occurs when

$$\frac{\sin \hat{i}_o}{\sin 90°} = n_{12}, \qquad \text{from which} \qquad \sin \hat{i}_o = \frac{n_2}{n_1} \ (n_1 > n_2). \tag{8.13}$$

In general at the interface between two media both reflection and refraction occour. The sum of reflected and refracted waves energies is the energy of the incident wave.

Reflection and refraction are used in the production of diagnostic images by means of ultrasounds (mechanical vibrations at ultrasonic frequencies); this use is treated in Chapter 12 (Paragraph 12.7).

Light is a specific example of an electromagnetic wave (Chapters 9 and 10). Electromagnetic waves do not need a medium in order to propagate. The speed of light in a vacuum, usually indicated by the letter $c$ (about $3 \cdot 10^5$ km/s), **is the maximum speed allowed in nature**. In the case of light, the index of absolute refraction in a vacuum is $n = 1$. The relative refractive index is expressed in relation to the absolute refractive index and is always greater than one (Chapter 10).

## 8.6 Wave interference

The word "**interference**" includes all those wave phenomena that arise from the superposition of two or more waves.

To simplify the treatment of the phenomenon, let us consider two waves with the same frequency $f$, equal amplitude $A$, the same direction of vibration (longitudinal or transversal) and coming from two coherent sources (**Figure 8.13**). Two sources are said to be **coherent** when they vibrate with a strictly **constant** phase difference.

At point P, the two vibrations are summed. From Eq. (8.5) we obtain:

$$S = S_1 + S_2 = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)\right] + A \sin\left[2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)\right]. \tag{8.14}$$

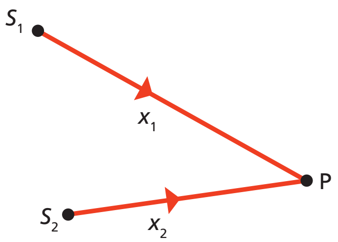



**Figure 8.13** The two vibrations add up at point P, giving rise to the phenomenon of interference.

The overall vibration at point P can be represented as a harmonic vibration with the same frequency $f$ of the components:

$$S = R \sin\left[\left(\frac{2\pi t}{T}\right) - \phi\right]. \tag{8.15}$$

Using the sum-to-product trigonometric formula Eq. (M1.17), we can obtain the amplitude $R$ and the phase $\phi$ of the resulting wave:

$$R = A\sqrt{2+2\cos\frac{2\pi(x_2 - x_1)}{\lambda}}, \tag{8.16}$$

$$\phi = \frac{\pi(x_2 + x_1)}{\lambda}. \tag{8.17}$$

The amplitude $R$ therefore depends on the phase difference between the two waves at the point where they meet. Specifically, when this difference is equal to zero or a multiple of $2\pi$, we have a resulting amplitude $R = 2A$ (**constructive interference**), while when it is equal to an odd multiple of $\pi$ the cosine in Eq. (8.16) is $-1$ and the amplitude $R = 0$ (**destructive interference**), as shown in **Figure 8.14**.

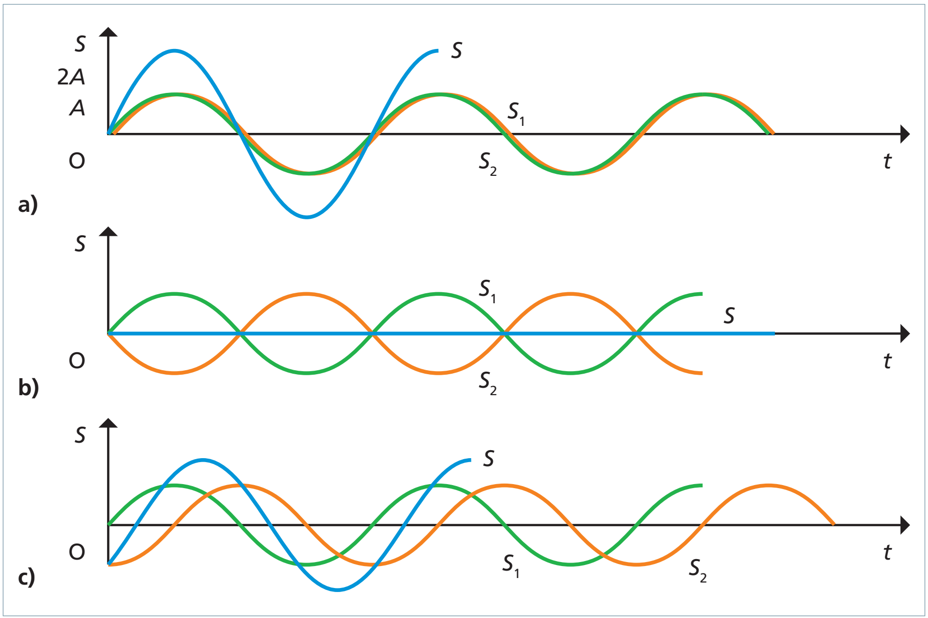



**Figure 8.14** **a)** The vibrations are **in phase**, the interference is constructive, and a vibration of the same frequency and double amplitude is generated (blue curve $S$). **b)** The vibrations are in **phase inversion** and the interference is destructive, resulting in an absence of vibration (if the amplitudes of the two waves are equal; blue line $S$). **c)** The vibrations are in **phase quadrature** and a vibration with the same frequency is generated, out of phase by $\pi/4$ and with an amplitude equal to $\sqrt{2}A$ (blue curve $S$).

To obtain the simple result of Eq. (8.15), it is mandatory the waves must be coherent. If the two vibrations do not have the same frequency, the result of their interference is more complicated. Two particular cases of interference are described below.

## 8.6.1 Standing waves

For simplicity, let us consider two waves with equal frequency (**mono-chromatic**), equal amplitude, generated at two points separated by a multiple of their wavelength, with zero phase difference $\Delta\phi$, and traveling on the same line, but in the opposite direction. Their sum in general is (applying the sum-to-product identity, Eq. (M1.17)):

$$S = S_1 + S_2 = A\sin\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right] + A\sin\left[2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)\right] = 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T}. \tag{8.18}$$

See **MATH INSET 1.2** Mathematical functions (p. 15)

The resulting wave is called a **standing wave**. It differs from a traveling wave (**Figure 8.15a**) since the space variable $x$ and time variable $t$ are separated in the arguments of the different sinusoidal functions. The standing wave is depicted in **Figure 8.15b**, in which there are fixed points where the amplitude is zero (**nodes**) and points where the amplitude reaches maximum values at different times (**antinodes**).

**Figure 8.16** shows some examples of standing waves along a string with fixed ends and in open-ended pipes: when persistent standing waves are formed in mechanical systems (strings, pipes or other more complex ones) the system is said to be in **resonance**. This effect occurs in wind or stringed musical instruments and also in the functioning of the basilar membrane in the inner ear (Paragraph 8.9.2).

## SOLVED PROBLEM 8.1

### Harmonic frequencies in the piano

Which are the frequencies of the first 3 harmonics of the longest string of a piano (length 1.98 meters) being the wave propagation speed on the string of 134 $\mathrm{m\,s^{-1}}$?

*Solution*

The string is fixed at both ends (see **Figure 8.16a**) and therefore the first harmonic has a wavelength equal to 2 $\ell$. The frequency for this harmonic is therefore:

$$f_1 = \frac{v}{2\ell} = \frac{134\ \mathrm{m\,s^{-1}}}{2 \cdot 1.98\ \mathrm{m}} = 33.84\ \mathrm{Hz}.$$

The second and third harmonic will be respectively $f_2 = 2 \cdot 33.84$ Hz = 67.68 Hz and $f_3 = 3 \cdot 33.84$ Hz = 101.52 Hz (all very low tones).

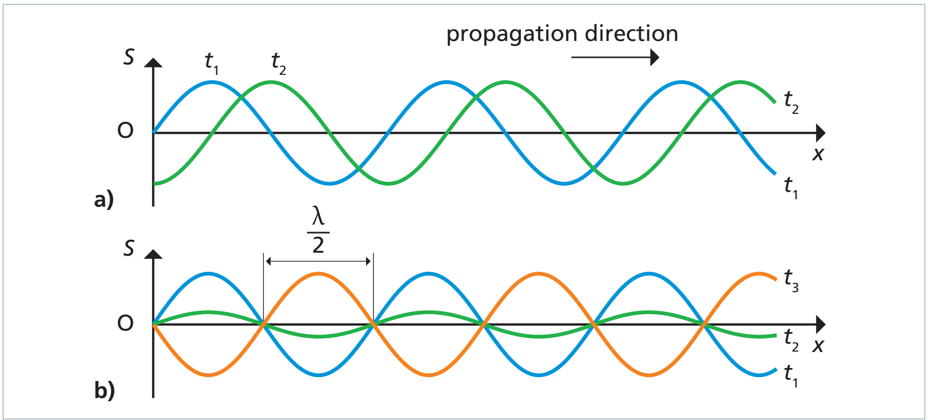



**Figure 8.15 a)** Traveling wave represented at successive times $t_1 > t_2$. **b)** Standing wave represented at successive times: $t_1 < t_2 < t_3$. At each point on the *x* coordinate there is a vibration with the same frequency; vhowever, its amplitude depends on the position on the *x* coordinate.

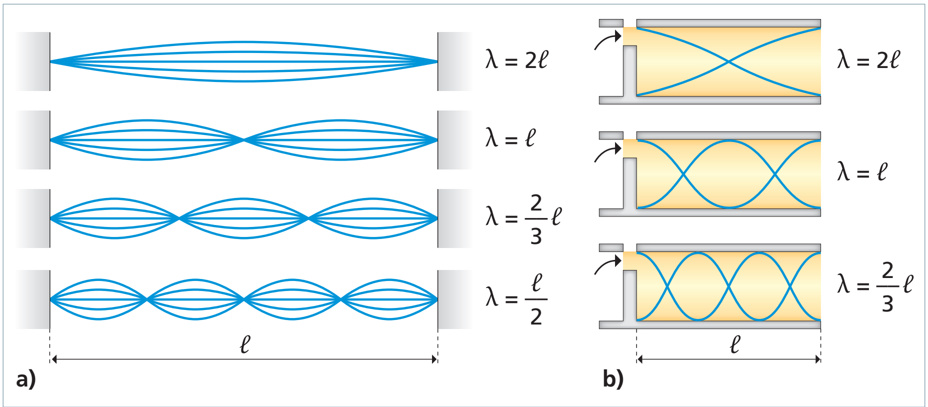



**Figure 8.16 a)** Standing waves on a string attached at both ends. The points of no displacement are nodes. **b)** The same phenomenon occurs in pipes, whether they are closed or open at the ends.

## 8.6.2 Beats

See MATH INSET 1.2
Mathematical functions (p. 15)

Now let us consider the superposition of waves with slightly different frequencies at fixed position. To simplify the calculations, we will consider two vibrations with zero phase and with the same amplitude. On calculating the sum and applying the appropriate trigonometric formulas (see Eq. (M1.17)), the resulting wave turns out to be

$$\begin{aligned} S &= S_1 + S_2 = A\sin(2\pi f_1 \cdot t) + A\sin(2\pi f_2 \cdot t) = \\ &= 2A\cos\left\{\frac{1}{2}[2\pi(f_1 - f_2)\cdot t]\right\}\cdot\sin\left\{\frac{1}{2}[2\pi(f_1 + f_2)\cdot t]\right\} \\ &= 2\,A\cos\left(\frac{2\pi\,\Delta f\cdot t}{2}\right)\cdot\sin\left[\frac{2\pi(f_1+f_2)\cdot t}{2}\right]. \end{aligned} \tag{8.19}$$

This result involves a sinusoidal vibration at an average frequency between the original frequencies $(f_1 + f_2)/2$, whose amplitude varies as the cosine with a frequency equal to half the difference $\Delta f/2$ between the two component vibrations. With two very close original frequencies the variation of the overall amplitude occurs very slowly (long period) even if with a frequency practically similar to the original ones (as shown in **Figure 8.17**).

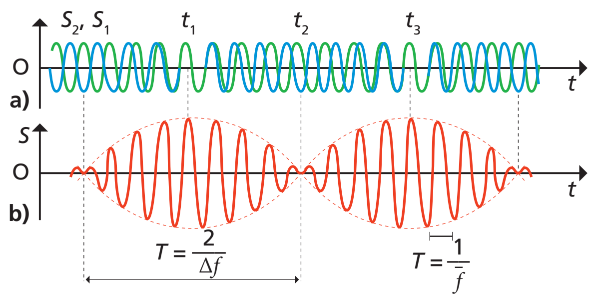



**Figure 8.17 a)** Two waves of slightly different frequency which are in phase at time $t_1$, in phase opposition at time $t_2$ and at time $t_3$ are again in phase agreement. **b)** Resultant wave of the two shown in **a)**. The frequency of the rapid oscillation is about the same as that of the original waves, but the amplitude is modulated and follows the dashed envelope with a very low frequency wave.

### SOLVED PROBLEM 8.2

#### Beats from a siren

A siren emits two sounds with frequencies $f_1$ = 1000 Hz and $f_2$ = 1004 Hz. Evaluate the resulting sound.

***Solution***

The two frequencies are almost equal and therefore we can apply the Eq. (8.19) which gives a frequency equal to the average value of the two frequencies:

$$\frac{1000\text{ Hz} + 1004\text{ Hz}}{2} = 1002\text{ Hz},$$

a fixed frequency practically indistinguishable from the initial frequencies. The amplitude of the sound at this frequency varies in a pulsatile manner (Eq. (8.19)):

$$\frac{1004\text{ Hz} - 1000\text{ Hz}}{2} = 2\text{ Hz},$$

with a periodicity of 0.5 second, therefore relatively slowly. The ambulance siren, the sound of which increases and decreases in intensity, is the result of a similar sum of sounds.

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## 8.7 Wave polarization

The phenomenon of **polarization** occurs **only** in the case of **transverse vibrations**. A transverse wave may be the result of the superposition of many waves, each with a random orientation of the vibration direction (**Figure 8.18a**). When the wave is **linearly polarized** (**Figure 8.18b**) the vibration occurs only in a specific direction. The plane in which the oscillation occurs is called the **plane of vibration**. When the wave is **circularly polarized**, the plane of vibration rotates with a constant angular velocity. Polarization of a wave can be obtained by means of specific filters.

In medicine polarization is mainly used in optical applications (see Paragraph 10.6).

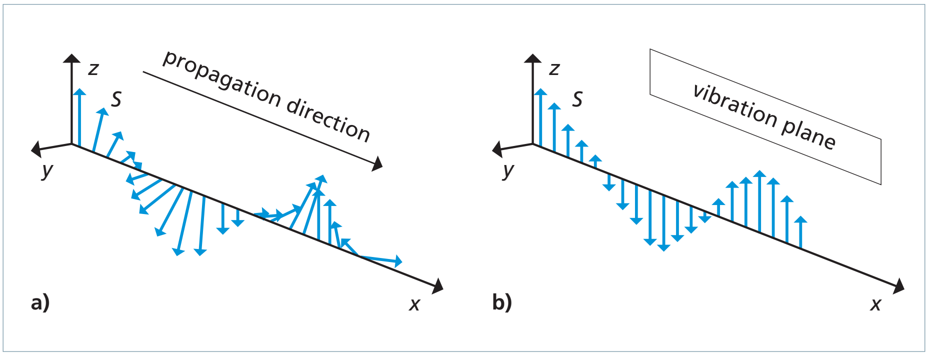


**Figure 8.18** Schematic representation of **a)** a non-polarized wave and **b)** a rectilinearly polarized wave. In both cases *S* is the physical vibrating quantity.

## 8.8 Doppler effect

When a source emits a wave of frequency $f_s$, the frequency $f$ that reaches a receiver depends on the state of motion of both the source and the receiver. This phenomenon is called the **Doppler effect** and has wide applications in diagnostic medicine (e.g., to measure blood velocity by means of ultrasound waves).

In the following, we will use $u$ to indicate the velocity of the receiver or of the source, and $v$ to indicate the propagation velocity of the wave in the medium. We will assume that the value of $u$ is always much lower than that of $v$.

Let us first consider the case in which the source S is stationary and the receiver O moves towards the source at a velocity $u_o$ (**Figure 8.19**). If the receiver were stationary, in one second it would detect a number of passing crests equal to the frequency $f_s$ included in the segment OR $= f_s\lambda$. If the receiver moved towards the source at speed $u_o$, it would detect a higher number of crests: not only the ones included in the segment OR, but also those included in the segment OR$_1 = u_o\Delta t$ ($\Delta t = 1$ s). In total, in one second it would receive all the waves contained in the RR$_1$ segment. The corresponding frequency (i.e., the number of crests detected per second), as in Eq. (8.6), is

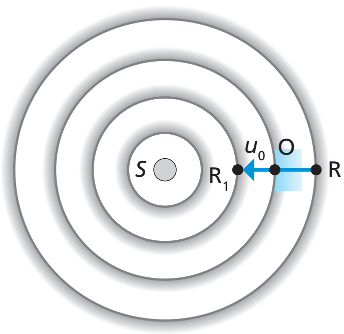


**Figure 8.19** Wavefronts emitted by a stationary source and received by an observer (point O) moving towards it. In time $\Delta t$, in addition to receiving the waves emitted by the source, the receiver also receives the waves contained in the segment OR$_1$, which travel in the same time interval $\Delta t$.

$$f_o = \frac{RR_1}{\lambda} = \frac{f_s\lambda + u_o}{\lambda} = f_s + \frac{u_o}{\lambda} = f_s + \frac{u_o f_s}{v} = f_s\left(1 + \frac{u_o}{v}\right), \tag{8.20}$$

the frequency $f_o$ perceived by the reciever is therefore greater than that emitted by the source $f_s$.

The change in frequency is

$$\Delta f = f_o - f_s = \frac{f_s u_o}{v}. \tag{8.21}$$

In the second case, the receiver is stationary and the source moves towards it with velocity $u_s$. Let S be the position of the source and O that of the observer (**Figure 8.20**). When the source has finished a complete vibration (i.e., after a period $T$), the point S has moved forward by a distance $u_s T$ and the wavelength $\lambda_o$ perceived by the observer will not be $\lambda_s$ but $\lambda_o = \lambda_s - u_s T$. Since $T = \lambda_s/v$ we therefore have

$$\lambda_o = \lambda_s - \frac{u_s \lambda_s}{v} = \lambda_s\left(1 - \frac{u_s}{v}\right). \tag{8.22}$$

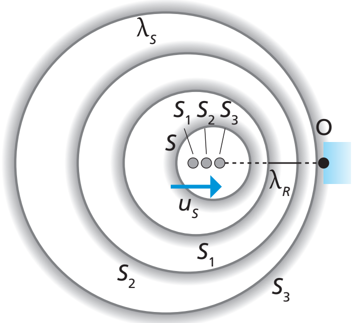


**Figure 8.20** Wave fronts emitted by a source moving at a speed $u_s$ towards a stationary receiver. The crests are closer together and the resulting wavelength is less than $\lambda_s$.

On expressing the wavelengths in terms of frequencies we have

$$f_o = f_s \frac{v}{v - u_s}. \tag{8.23}$$

The change in frequency in this case is given by

$$\Delta f = f_o - f_s = f_s \frac{u_s}{v - u_s}. \tag{8.24}$$

Ultimately, in both cases, the frequency received by the observer is greater than that emitted by the source. In the case of sound waves, if the source and/or receiver move towards each other, the perceived sound has a higher pitch.

If the receiver moves away from the fixed source or the source moves away from the stationary receiver, the signs of the $u_o$ and $u_s$ velocities in the above relations must be changed and, in both cases, the frequency received by the observer decreases.

When both source and receiver are moving, the general formula is

$$f_o = \frac{1 + \dfrac{u_o}{v}}{1 - \dfrac{u_s}{v}} f_s. \tag{8.25}$$

## SOLVED PROBLEM 8.3

### Distance and velocity from echo

A bat moves towards a stationary obstacle emitting sounds at the frequency of $50 \cdot 10^3$ Hz and detects, after 0.06 s, a reflected sound at the frequency of $51 \cdot 10^3$ Hz. How far is it from the obstacle and at what speed is the bat moving? (Assume sound velocity: 340 m s$^{-1}$.)

***Solution***

We have a moving source (the bat) with a stationary obstacle (a wall): the frequency change between the obstacle and the source $f_o - f_s$ caused from the Doppler effect is given by Eq. (8.21):

$$f_o - f_s = f_s \frac{u}{v}.$$

The ultrasound wave is reflected by the obstacle at a frequency $f_o$ and is detected by the approaching bat at a frequency $f_r$. In this case, the frequency change $f_r - f_o$ is

**SOLVED PROBLEM 8.3**

given by Eq. (8.24):

$$f_r - f_o = f_o \frac{u}{u+v}.$$

The total change $f_r - f_s$ is obtained eliminating $f_o$ in these two equations (deriving $f_o$ from the first equation and introducing it in the second one), with the result

$$f_r - f_s = f_s \frac{2u}{v},$$

from which we have

$$u = \frac{(f_r - f_s)v}{2 f_s} = \frac{(51 \cdot 10^3 - 50 \cdot 10^3)\ \text{Hz} \times 340\ \text{m s}^{-1}}{2 \times 50 \cdot 10^3} = 3.4\ \text{m s}^{-1} = 12.24\ \text{km h}^{-1}.$$

The ultrasound wave at the speed of 340 m s$^{-1}$ travels in 0.06 seconds a distance of $d = v\Delta t = 340\ \text{m s}^{-1} \cdot 0.06\ \text{s} = 20.4$ meters; as it has to travel from the bat to the obstacle and then back to the bat, the obstacle is located at a distance of 20.4/2 = 10.2 meters.

As we have already mentioned, light is a form of wave and, as such, it is also subject to the Doppler effect. The correct formula for light (**relativistic Doppler effect**) is

$$f_o = f_s \sqrt{\frac{1+v/c}{1-v/c}}, \tag{8.26}$$

where $c$ is the speed of light and $v$ is the relative velocity between the source and the observer (it does not matter which of the two is moving). In the case of light, too, if the source and/or receiver move towards each other (positive $v$), the frequency of the light wave is higher. This means that the observed light has a different color (see Paragraph 9.9 and Paragraph 10.2).

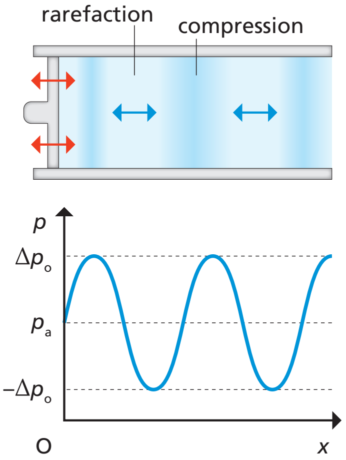


Figure 8.21 Schematic figure of areas of compression and rarefaction. In the propagation of a sound wave in a gas, pressure variations are caused by the harmonic motion of the gas molecules. This is caused by the piston moving back and forth in harmonic motion.

## 8.9 Sound waves, infrasounds and ultrasounds

### 8.9.1 Sound waves

Small deformations in an elastic medium produce reaction forces that tend to bring the medium back to its equilibrium configuration. If a region of the medium is compressed (e.g., by the action of the oscillating membrane of a loudspeaker), reaction forces will cause it to expand, compressing the neighboring regions (**Figure 8.21**). The resulting propagating disturbance is a **sound** (also called **acoustic**) **wave**.

Sound waves are therefore elastic mechanical waves in a medium. If the medium is a gas or a liquid, the sound waves are longitudinal pressure waves. In the case of solids, sound waves can be both longitudinal (pressure waves) and transverse (shear waves).

The human ear is able to perceive acoustic waves propagating in air at frequencies between 20 Hz and $2 \cdot 10^4$ Hz. Below 20 Hz, these vibrations are called **infrasounds**, above $2 \cdot 10^4$ Hz, they are called **ultrasounds**.

**Pure sounds** are simple harmonic vibrations, while complex sounds are superpositions of harmonic waves.

The **pitch** of a sound is proportional to its frequency. The **timbre** of a sound depends on the shape of the vibration and therefore on the amplitude of its harmonic components, as shown by Fourier analysis.

See MATH INSET 8.1 Fourier (or harmonic) analysis (p. 182)

The **sound intensity**, defined in general terms in Paragraph 8.4, depends on the energy carried by the sound wave; from Eqs. (M8.2), (M8.4) and (8.10), this depends on the sum of the squares of the amplitudes of the simple vibrations that make up the complex sound.

A harmonic sound wave can be described by the formula

$$\Delta p(t) = \Delta p_o \sin(\omega t + \phi), \tag{8.27}$$

where $\Delta p(t) = p - p_a$ is the instantaneous variation of the pressure $p$ with respect to the atmospheric pressure $p_a$ and $\Delta p_o$ is the amplitude of the pressure perturbation.

On applying the instantaneous sound pressure (Eq. (8.27)) to the piston in **Figure 8.22**, we see that the air mass in the pipe is set in oscillatory motion. By means of the second law of dynamics (Eq. 1.17), we obtain

$$\Delta p\, S = \frac{mu}{\Delta t}, \tag{8.28}$$

where $u$ is the instantaneous speed of the piston with a surface $S$. By introducing the density of the gas $d = m/V = m/(S\Delta\ell)$ ($\Delta\ell$ is the length of the pipe containing the mass $m$ of gas), we obtain

$$\Delta p = \frac{u\, d\, \Delta\ell}{\Delta t} = u\, d\, v, \tag{8.29}$$

where $v = \Delta\ell/\Delta t$ is the velocity of the wave in the gas.

If $A$ is the amplitude of the oscillatory motion of the piston, the velocity $u = u(t)$ is sinusoidal with amplitude $A \cdot \omega$ (see Eqs. (1.14) and (8.9)); we therefore obtain the following relationship between the amplitudes of Eq. (8.27) and Eq. (8.29):

$$\Delta p_o = A\, \omega\, d\, v. \tag{8.30}$$

Notice that the change of pressure in a sound wave is proportional to the density of the medium and to the velocity of sound in that medium.

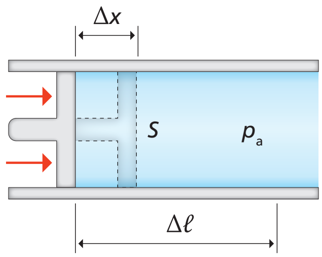


**Figure 8.22** Air at atmospheric pressure $p_a$ inside a pipe is compressed by a piston with a surface S, giving rise to a sound pressure wave.

## 8.9.2 Sound wave velocity and intensity

If a sound wave propagates in an ideal gas, its velocity $v$ depends only on the absolute temperature $T$:

$$v = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{\gamma p_a}{d}}, \tag{8.31}$$

where $\gamma$ is the $c_p/c_v$ ratio, $R$ the constant of the ideal gases and $M$ the molecular weight of the gas. Sound travels faster in a gas at higher temperature, higher pressure, or lower density.

Furthermore on applying Eq. (8.10) to a sound wave traveling through the volume shown in **Figure 8.23**, the energy transported is

$$E = \frac{1}{2}\omega^2 m A^2 = \frac{1}{2}\omega^2 V d A^2 = \frac{1}{2}\omega^2 S v \Delta t\, d A^2. \tag{8.32}$$

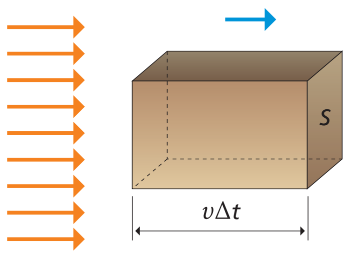


**Figure 8.23** A sound wave propagating through a surface $S$ at velocity $v$ travels a longitudinal distance $v\Delta t$ in the time interval $\Delta t$. The volume through which it travels is therefore $S v \Delta t$.

On dividing Eq. (8.32) by $S\Delta t$, we obtain the intensity of the sound wave $I$ defined in Paragraph 8.4:

$$I = \frac{E}{S\Delta t} = \frac{1}{2} v d \omega^2 A^2, \tag{8.33}$$

which we can also write in terms of pressure amplitude by using Eq. (8.30):

$$I = \frac{1}{2}\frac{\Delta p_o^2}{vd} \tag{8.34}$$

from which we obtain the relationship between sound intensity and sound pressure amplitude

$$\Delta p_o = \sqrt{2Ivd}. \tag{8.35}$$

## SOLVED PROBLEM 8.4

### Sound velocity in Helium

Evaluate the velocity of sound in helium at 32 °C. The helium molecular weight is 4 and the $\gamma$ ratio is 1.66.

#### *Solution*

On applying Eq. (8.31), and remembering that the Boltzmann constant $k_B = R/N_o$ (see Paragraph 5.5) and the mass of a molecule is given by its molecular weight divided by Avogadro's number, we have

$$v = \sqrt{\frac{\gamma k_B T}{m}} = \sqrt{\frac{1.66 \cdot 1.38 \cdot 10^{-23}\ \mathrm{JK^{-1}} \cdot 305\ \mathrm{K}}{4 \cdot 10^{-3}\ \mathrm{kg} \cdot (6.02 \cdot 10^{23})^{-1}}} = 1025.4\ \mathrm{m\,s^{-1}}$$

This is almost three times the speed of sound in air. Breathing a mixture of air in which nitrogen has been replaced by helium has a weird effect on our voice. Sound is produced by the vocal cords in our voice box, and then travels to the mouth. The shape of the mouth amplifies certain frequencies and plays a major role in determining the timbre of our voice. Since sound travels faster in helium than in air, the mouth amplifies higher frequencies than normal, altering the timbre of our voice and making it sound squeaky.

## 8.9.3 Sound frequency spectrum

**Figure 8.24** shows the spectrum of sound waves. **Infrasounds** are vibrations with a frequency lower than 20 Hz. These vibrations are characterized by the ability to propagate over long distances and to circumvent obstacles without dissipating much energy.

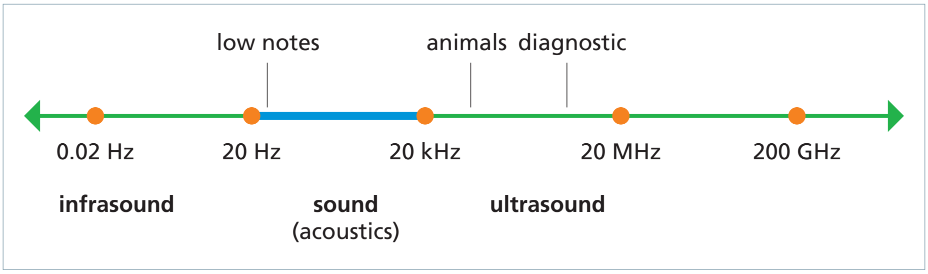


**Figure 8.24** Frequency spectrum of mechanical waves: the frequency scale is logarithmic. The range of frequencies perceptible to the human ear is shown in blue. **Acoustics** describes the characteristics of sounds.

**Ultrasounds** have frequencies above $2 \cdot 10^4$ Hz and are artificially produced by means of particular **piezoelectric crystals**, which can also act as ultrasound detectors. Piezoelectric crystals can be rapidly deformed by the application of an electric voltage and conversely produce an electric voltage when deformed.

As explained in Paragraph 8.4, a very important feature of all kinds of vibration is their **directionality**, which is **proportional to their frequency**. This implies that at high frequency (1÷10 MHz and beyond) ultrasound propagates as a very collimated sound beam.

**SOLVED PROBLEM 8.5**

**Infrasound produced by sea waves**

A cork floating in the sea completes one oscillation every 5 seconds: evaluate its frequency. If the distance between two successive wave crests is 4 meters, evaluate their propagation velocity.

***Solution***

The inverse of the period gives the resulting frequency:

$$f = T^{-1} = \frac{1}{5\ \text{s}} = 0.20\ \text{Hz}.$$

The frequency is therefore that of an infrasound. The wave velocity is given by the relation

$$v = \lambda f = 4\ \text{m} \cdot 0.20\ \text{s}^{-1} = 0.8\ \text{m s}^{-1} = 2.88\ \text{km/h}.$$

As the sea wave propagates, it acts on the molecules of the air, causing an infrasonic pressure vibration that is not audible by human hearing in terms of frequency and intensity. Infrasonic vibrations of high intensity are easily perceived in the disco as pressure waves on the chest.

# 8.10 Sound sensation and the ear

## 8.10.1 Sound sensation

In humans (and in other animals) sound is detected by the ear, which we will describe in the next paragraph. First, we need to discuss the concepts of sound intensity, intensity level and loudness.

The ear is a very sensitive device. For example, during a conversation of medium intensity, as reported in **Table 8.1**, we have $I = 10^{-6}$ W/m², $d = 1.2$ kg/m³, $v = 340$ m/s: from the relation (8.35) we have a variation in pressure on the eardrum in the order of $\Delta p_o = 3 \cdot 10^{-2}$ N/m².

Since the value of atmospheric pressure is 1 atmosphere = $10^5$ N/m², it follows that, during the conversation, the human ear is able to perceive pressure fluctuations in the order of 1 part in 10 million!

Taking **Table 8.1** as a reference, the most violent sound that the human ear can tolerate is about 10 N/m² ($10^{-4}$ atmospheres), while the hearing threshold is about $10^{-5}$ N/m² (= $10^{-10}$ atmospheres); this means that the human ear can perceive pressure variations of 1 part in 10 billion! It is interesting that the collisions of air molecules on the eardrum membrane due to thermal agitation cause pressure variations of about $10^{-11}$ atmospheres, only a tenth of the hearing threshold.

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In conclusion, the ear is able to perceive sound pressure differences in the order of one part in $10^{10}$ and the extension of the sound pressure amplitude covers 6 orders of magnitude. This enormous extension imposes limitations on the auditory system's ability to distinguish between different sound pressure vibrations.

The relationship between sound intensity and perceived **sound sensation** $\sigma$ is logarithmic: two sound intensities are perceived as different only if the difference exceeds a fixed minimum value, which is proportional to the intensity $I$:

$$\Delta I \geq \varepsilon I, \tag{8.36}$$

where $\varepsilon = 10^{-1}$. This inequality, known as **Weber-Fechner law**, expresses the fact that the human ear is capable of appreciating variations in sound intensity that are smaller than the lower sound intensity $I$. From these considerations it seems logical to assume the following relationship between the minimum variation in sound sensation $\sigma$ and the minimum variation in sound intensity $I$:

$$\Delta I = \Delta\sigma I, \qquad \text{from which} \qquad \Delta\sigma = \frac{\Delta I}{I}, \tag{8.37}$$

which, considering $\Delta I$ and $\Delta\sigma$ as differentials and integrating them, yelds the **sound intensity level** IL:

$$\mathrm{IL} = \sigma - \sigma_o = 10\ \mathrm{Log}\frac{I}{I_o}, \tag{8.38}$$

where a base-10 logarithm is introduced and the factor 10 is included for convenience and convention. The $I_o$ value represents the minimum sound intensity that can be detected by the human ear and corresponds to $10^{-12}$ W/m$^2$ (from Eq. (8.38) this corresponds to sound intensity level IL = 0 and $\sigma_o$ is the value of sound sensation when $I = I_o$). The unit of measurement of sound intensity level expressed by Eq. (8.38) is the **decibel** (dB) (the official unit of measure is the **bel**, corresponding to 10 dB). The sound intensity level is usually expressed in decibels as shown in **Table 8.1**.

The sound intensity level is related to the intensity of the sound wave, independently of its frequency. However, the human ear is more sensitive to sounds with frequencies between 3000 and 4000 Hz. To describe this phenomenon we use the concept of **loudness**. **Figure 8.25** shows curves of equal loudness as a function of frequency. A 70 dB sound at 4000 Hz is perceived as loud as a 90 dB sound at 60 Hz. The unit of measurement of loudness is the **phon**: a sound with a loudness of 10 phon is as loud as a sound with an intensity level of 10 dB at 1000 Hz.

**Table 8.1** Sound intensity and sound sensation.

| Sources | $I$ (watt m$^{-2}$) | $\sigma$ (dB) |
|---|---|---|
| whisper at 1 m distance | $10^{-10}$ | 20 |
| average home noise | $10^{-8}$ | 40 |
| normal conversation | $10^{-6}$ | 60 |
| scream | $10^{-5}$ | 70 |
| noisy traffic | $10^{-4}$ | 80 |
| siren at 30 m | $10^{-2}$ | 100 |
| pneumatic drill at 1 meter (threshold of pain) | 1 | 120 |
| jet airplane at 30 m (severe pain) | $10^{2}$ | 140 |
| bursting of eardrums | $10^{4}$ | 160 |

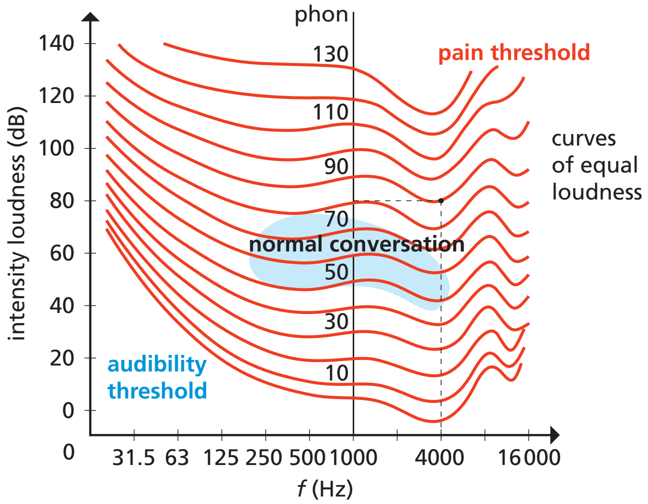


**Figure 8.25** Curves of equal loudness as a function of frequency. A sound with intensity level 80 dB at 1000 Hz has a loudness of 80 phon. A sound with the same intensity level at 4000 Hz has a loudness of 90 phon, because the human ear is more sensitive at that frequency.

## SOLVED PROBLEM 8.6

### Sound intensity from a jet at high altitude

A jet flying at an altitude of 4000 meters produces a sound of 40 dB on the ground. Calculate the sound intensity level if the altitude was 500 meters (assume that the total sound energy produced does not change).

#### *Solution*

From Eq. (8.38), 40 dB gives: $I_{4000} = 10^4 \cdot I_o$. The sound intensity at 500 m is (from Paragraph 8.4): $I_{500} = (4000/500)^2 \cdot 10^4 \cdot I_o = 64 \cdot 10^4 \cdot I_o$, so the sound intensity level at this altitude, according to Eq. (8.38), is

$$\text{IL} = 10 \,\text{Log}[64 \cdot 10^4] = 58.06 \text{ dB}.$$

## SOLVED PROBLEM 8.7

### Maximum tolerable sound pressure

The maximum amplitude $\Delta p_o$ of a sound wave that is still tolerable by the human ear is about 28.5 pascal. (1) What fraction of normal atmospheric pressure at sea level does this value correspond to? (2) To what sound intensity does this $\Delta p_o$ value in air corresponds, assuming an air density of 1.29 $\text{kg}\,\text{m}^{-3}$ and a speed of sound of 340 $\text{m}\,\text{s}^{-1}$?

#### *Solution*

(1) Since the normal atmospheric pressure is 1 atm = $1.01 \cdot 10^6$ barie = $1.01 \cdot 10^5$ Pa, we have

$$\frac{\Delta p_o}{p} = \frac{28.5 \text{ Pa}}{1.01 \cdot 10^5 \text{ Pa}} = 2.82 \cdot 10^{-4}.$$

Thus, even very loud sounds correspond to pressure fluctuations which are a very small fraction of the atmospheric pressure.

(2) On applying Eq. (8.34) we obtain intensity $I$:

$$I = \frac{\Delta p_o^2}{2dv} = \frac{(28.5 \text{ Pa})^2}{2 \cdot 1.29 \text{ kg}\,\text{m}^{-3} \cdot 340 \text{ m}\,\text{s}^{-1}} = 0.926 \text{ W}\,\text{m}^{-2}.$$

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## 8.10.2 Ear and sound transmission

The **ear** is the transduction device that enables sound waves to be transformed into signals of nervous excitation (action potentials), which are then processed and organized by the brain in order to provide understandable sound sensations (spoken language, music, environmental sounds and so on). From a functional point of view, the ear can be divided into three successive parts: external, middle and internal (**Figure 8.26**).

The **external ear** is made up of the pinna and the auditory canal, which collect sound and channel it towards the eardrum. The auditory canal is, in practice, a resonator and can be schematized as a tube closed at one end by the tympanic membrane (see **Figure 8.16b**). It is therefore the site of stationary waves (Paragraph 8.6.1), allowing the mechanical vibration to be maintained over time. The fundamental frequency in adults is between 3000 and 4000 Hz (see Solved problem 8.8), in agreement with the loudness diagram in **Figure 8.25**. If the tympanic membrane were rigid, the auditory canal would only resonate at the frequency of about 3400 Hz or **odd multiples** of this value. In reality, the membrane is elastic enough to cause resonances at lower and higher frequencies, as shown in **Figure 8.27**.

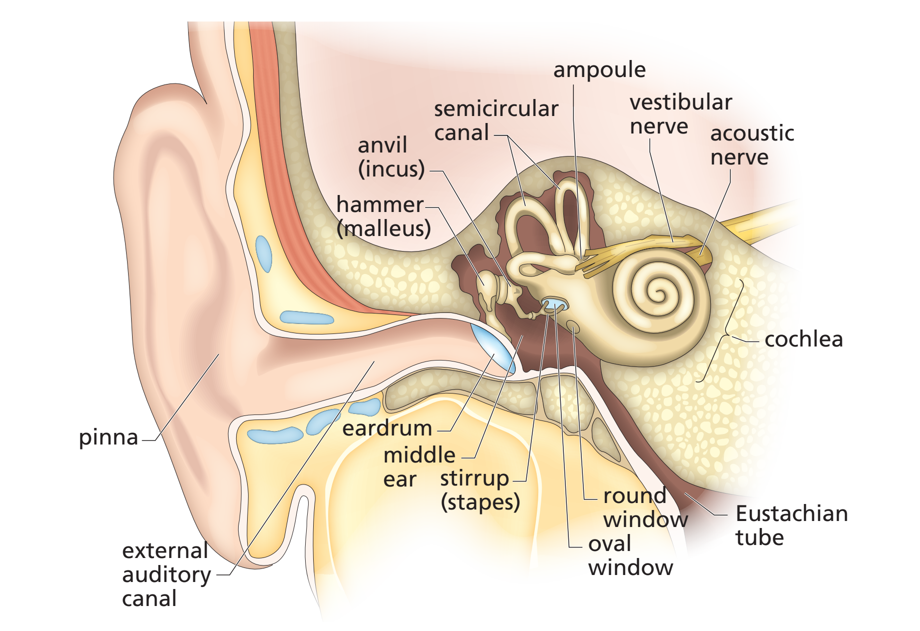


**Figure 8.26** Schematic representation of the anatomy of the human ear. Observe the three-dimensional helical structure of the cochlea. The figure also shows the semicircular canals involved in the sense of balance.

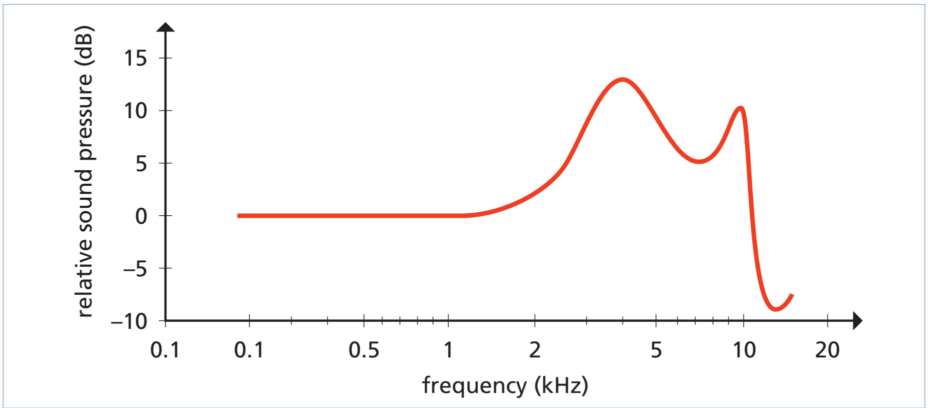


**Figure 8.27** At a frequency around 3400 Hz there is a peak in the frequency response of the ear. A secondary peak occurs near 10 000 Hz (about $3 \cdot 3400$ Hz). The first peak indicates that the ear is specifically tuned to the human voice.

## SOLVED PROBLEM 8.8

### Harmonic frequencies in tubes

Calculate the fundamental frequency and the first two successive harmonics in two tubes: (1) one closed at both ends and (2) one closed at only one end. Both tubes are 7.7 cm long and the sound velocity is 340 m s$^{-1}$.

#### *Solution*

(1) The first harmonic has a wavelength that is twice the length of the tube (**Figure 8.16**); the corresponding frequency is 340 m s$^{-1}$/0.154 m = 2207.8 Hz and the successive harmonics are $f_2$ = 4415.6 Hz and $f_3$ = 6623.4 Hz.

(2) The first harmonic has a wavelength 4 times the length of the tube (**Figure 8.16**); the corresponding frequency is 340 m s$^{-1}$/0.308 m = 1103.9 Hz and the successive harmonics are $f_2$ = 2207.7 Hz and $f_3$ = 3311.7 Hz.

## SOLVED PROBLEM 8.9

### Ear canal harmonic frequencies

Calculate the fundamental harmonic frequency in an auditory canal about 2.6 cm long in an adult subject. The sound velocity is 340 m s$^{-1}$.

#### *Solution*

The auditory canal is closed by the eardrum: we therefore have a tube that is open at one end and closed at the other. The wavelength of the first harmonic is

$$\lambda = 4 \cdot 2.6 \text{ cm} = 10.4 \text{ cm}.$$

The corresponding frequency is

$$f = \frac{v}{\lambda} = \frac{340 \text{ m s}^{-1}}{0.104 \text{ m}} = 3269.2 \text{ Hz},$$

which is in the range of frequencies of spoken language.

In the **middle ear**, the function of the three **ossicles (malleus, incus, and stapes)** is to transmit the sound vibration to the **oval window** by **amplifying** it. The ossicles act as an advantageous lever of the 1st type in which the distance $\delta_i$ between the eardrum and the fulcrum is approximately 1.3 times the distance $\delta_f$ between the oval window and the fulcrum (**Figure 8.28**).

The equilibrium condition described in Eq. (2.9) becomes

$$\frac{F_f}{F_i} = \frac{\delta_i}{\delta_f} = 1.3, \qquad (8.39)$$

from which, as the area of the eardrum $S_i$ is about 20 times greater than that $S_f$ of the oval window, we obtain a pressure amplification factor on the oval window of

$$\frac{p_f}{p_i} = \frac{F_f S_i}{F_i S_f} = \frac{\delta_i S_i}{\delta_f S_f} = 1{,}3 \cdot 20 = 26. \qquad (8.40)$$

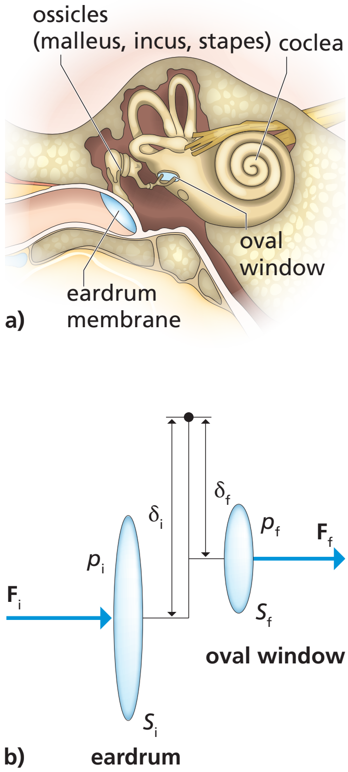


**Figure 8.28 a)** The action of the three ossicles—hammer (malleus), anvil (incus), stirrup (stapes)—of the middle ear is to amplify the pressure from the eardrum to the oval window. **b)** Diagram of the advantageous 1st type lever in the middle ear.

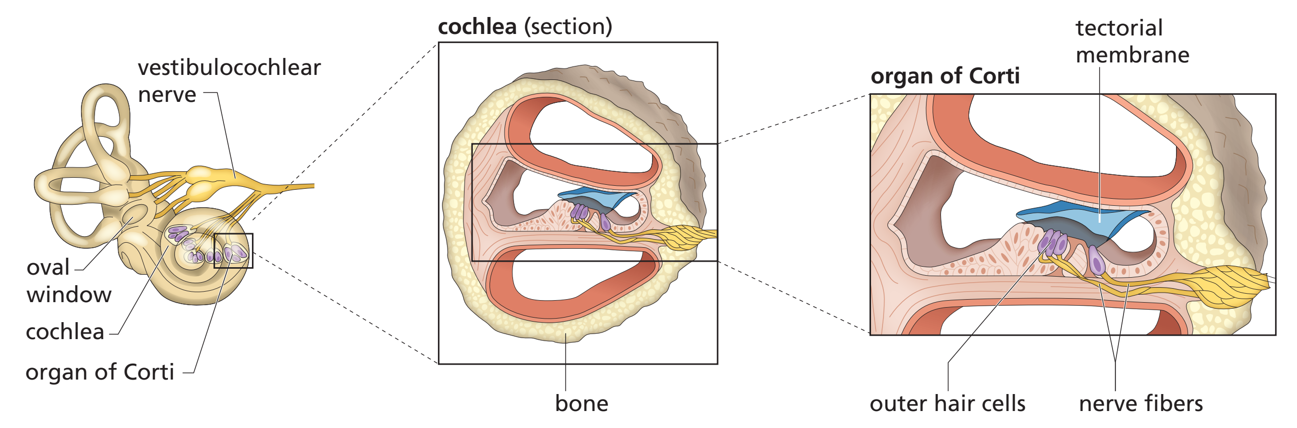


**Figure 8.29** Anatomical section of the human cochlea with subsequent enlargements of its parts. The vestibular duct (scala vestibuli) and tympanic duct (scala tympani) are filled with perilymph, while the cochlear duct contains endolymph. Hair cells with stereocilia are arranged between the two membranes, basilar and tectorial (note their microscopic dimensions). The stereocilia can be mechanically excited at different frequencies according to their length (Paragraph 8.6). Their excitation, determined by the acoustic wave propagating in the perilymph and in the endolymph, generates action potentials in the cells of the acoustic nerve.

This amplification factor is necessary in order to compensate for the loss of sound intensity that would otherwise occur when passing from air in middle ear to liquid in the inner ear cochlea.

Finally, the **inner ear** is made up of a spiral-shaped canal about 3.5 cm long (**cochlea**). Inside it, (**Figure 8.29**) and along its entire length, there is a membrane (**basilar membrane**) on which the organ of Corti is located; this enables mechanical vibrations to be transduced into action potentials.

If we assume that the intensity of the sound wave (at a given frequency) arriving at the oval window $I_1$ and that of the sound wave propagating from the oval window to the cochlear fluid (perilymph) $I_2$ remain constant, on applying Eq. (8.33) we have

$$I_1 = \frac{1}{2} d_1 A_1{}^2 v_1 \omega^2 = I_2 = \frac{1}{2} d_2 A_2^2 v_2 \omega^2, \tag{8.41}$$

where $d_1$ and $d_2$ are the densities of the two media (air and perilymph), $A_1$ and $A_2$ are the amplitudes of the sound vibration in these media, and $v_1$ and $v_2$ are the velocities of the vibration, respectively ($d_2 = 1000\ \mathrm{kgm^{-3}}$, $v_2$ (water) $= 1527\ \mathrm{m\,s^{-1}}$, Table 12.4). Therefore, the ratio between the amplitudes of vibration in the two media must be

$$\frac{A_1}{A_2} = \sqrt{\frac{d_2 v_2}{d_1 v_1}} = \sqrt{\frac{1000\ \mathrm{kg\,m^{-3}}}{1.2\ \mathrm{kg\,m^{-3}}} \frac{1527\ \mathrm{m\,s^{-1}}}{340\ \mathrm{m\,s^{-1}}}} \approx 60. \tag{8.42}$$

This damping factor is partially compensated for by amplification in the middle ear, described in Eq. (8.40). Technically, we say that the impedance of the air-to-perilymph interface is high, and that the ossicular system of the middle ear acts as an **impedance adapter**[1].

[1] Like mechanical impedance (Paragraph 4.10) and electrical impedance (Paragraph 9.7), acoustic impedance can be defined by referring to the transmission of sound waves in free media or through the interface between two different media, as described in Paragraph 12.7.1.

As a sound wave enters the cochlea and propagates through the perilymph, the basilar membrane vibrates. Depending on the frequency of the sound, the amplitude of the vibration of the membrane has a maximum value at different positions (closer to the beginning of the cochlea for higher frequencies, further inside the cochlea for lower frequencies).

In the inner ear, inside the organ of Corti, there are **hair cells** with microscopic protuberances (**stereocilia**). From Helmoltz theory the stereocilia resonate according to their tension and length as fixed strings at the ends (see Paragraph 8.6.1). The movement of the basilar membrane causes the sterocilia to brush against the tectorial membrane, stimulating various nerve fibers, which transmit action potentials (see Paragraph 7.13 and Paragraph 7.14) to the brain via the cochlear nerve.

## 8.11 Stethoscope and body sounds

The **stethoscope**, or **phonendoscope**, is perhaps the most famous symbol of medical professionals. This acoustic device allows doctors to listen to the internal sounds produced by the human body, especially those coming from the heart and lungs. The forerunner of the modern stethoscope was invented by the French physician R. T. H. Laënnec in 1818.

This instrument (**Figure 8.30**) consists of a headset with two eartips, two metal tubes connected to a flexible tube, and a chestpiece. Modern devices usually have a dual-head chestpiece with an **open bell** and a **drum**, i.e., a bell closed by a thin diaphragm. The open bell is more sensitive to low-frequency sounds, while the drum is used for high-frequency sounds, as shown in **Figure 8.31**. The frequency of maximum sensitivity of the drum depends on the thickness and material of the diaphragm. In the case of the open bell, sensitivity depends on its shape and on how firmly it is pressed against the skin, which effectively acts as a diaphragm.

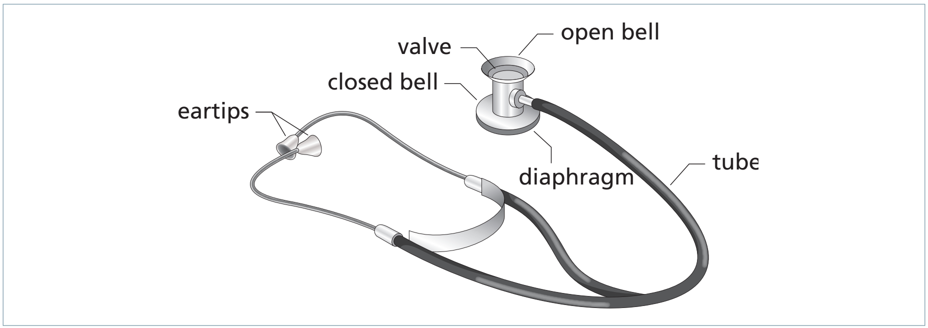


**Figure 8.30** A modern stethoscope.

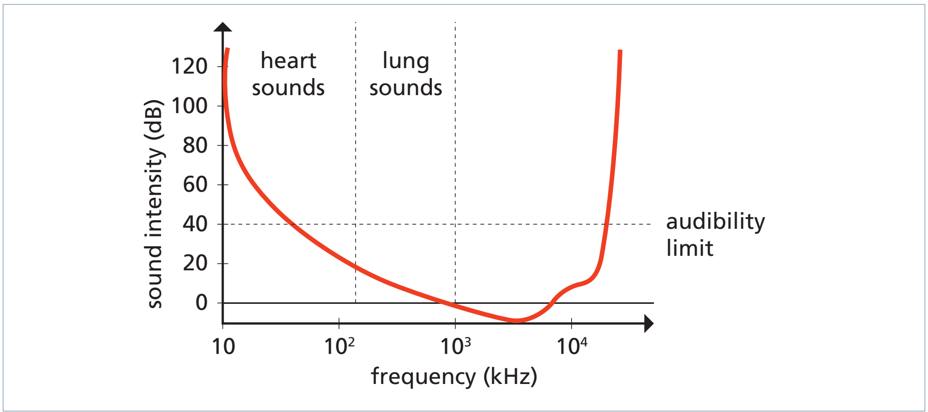


**Figure 8.31** Most heart sounds are of low frequency, in the low sensitivity range. Conversely, lung sounds generally have a higher frequency. Some heart and lung sounds have an intensity below the audible threshold, as shown in the figure.

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# 8.12 Ultrasound in medicine (diagnostic and therapy)

Ultrasonic vibrations are exploited in several medical applications. One of their main advantages is that ultrasound rays are highly collimated, owing to their high frequency, higher than one megahertz, as described at the end of Paragraph 8.9. For this reason, ultrasounds are used in **sonography** (i.e., **ultrasound scans**), as described in detail in Chapter 12 (Paragraph 12.7). In the remainder of this paragraph, we will describe an important diagnostic application of ultrasounds (Doppler flowmetry) and mention some of their many therapeutic uses.

## 8.12.1 Doppler flowmetry

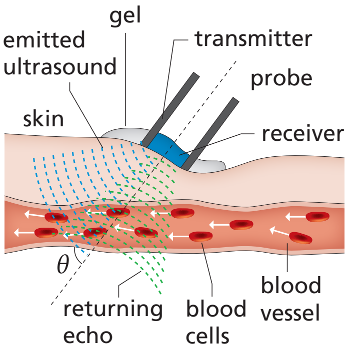


**Figure 8.32** The figure shows schematically how the Doppler flowmeter works. The ultrasonic beam emitted by the source is reflected by the surface of the moving erythrocytes. Owing to the Doppler effect, the frequency of the returning echo is different from that of the emitted sound. The device (probe) contains two piezoelectric crystals, one transmitting and one receiving.

The **Doppler flowmeter**, shown in **Figure 8.32**, allows the measurement of the instantaneous velocity of blood. It consists of a probe that contains a piezoelectric transmitter (source), which produces an ultrasound wave with frequency $f_s$ aimed at the vessel in which the blood flow is to be measured. The ultrasound from the transmitter is reflected by the red blood cells, which are moving at velocity $u$, and is detected by a piezoelectric receiver contained in the same probe.

The probe is held at an angle $\theta$ to the vessel axis (**Figure 8.32**). The component of the velocity of the blood cells in the direction of the probe is

$$u = -\, u_\theta = (-u\cos\theta) \tag{8.43}$$

(the minus sign is due to the fact that in **Figure 8.32** the blood cells are moving away from the probe). The red blood cells act first as receivers of the sound produced by the transducer, then as emitters of a reflected sound.

Since they are moving in relation to the source, owing to the Doppler effect the red blood cells perceive a frequency of the ultrasound given by the relation (8.20) which we rewrite here as

$$f_i = f_s\left(1 - \frac{u_\theta}{v}\right), \tag{8.44}$$

where $v$ is the speed of sound in the blood. The red blood cells then reflect the ultrasounds, acting as moving sources that emit a frequency $f_i$. A receiving piezoelectric transducer (stationary observer) measures the frequency of the reflected ultrasound $f_r$, which is again changed by the Doppler effect, as given by Eq. (8.23)

$$\begin{aligned} f_r &= f_i\frac{v}{v+u_\theta} = f_s\frac{v-u_\theta}{v}\frac{v}{v+u_\theta} = f_s\frac{v-u_\theta}{v+u_\theta} = \\ &= f_s\frac{v-u_\theta+u_\theta-u_\theta}{v+u_\theta} = f_s\left(1-\frac{2u_\theta}{v+u_\theta}\right). \end{aligned} \tag{8.45}$$

We can then obtain the difference in frequency between the emitted ultrasound and the received ultrasound:

$$\Delta f = f_s - f_r = f_s\frac{2u_\theta}{v+u_\theta}. \tag{8.46}$$

In conclusion, by measuring the frequency change it is possible to deduce the instantaneous velocity $u$ of the blood in the vessel (the sound velocity $v$ is known, in practice the same as in water). Since the blood velocity is much lower than the sound velocity ($u << v$), $v + u_\theta$ is approximately equal to $v$, and Eq. (8.46) becomes

$$\Delta f = f_s - f_r \approx f_s \frac{2u}{v}, \tag{8.47}$$

This is the formula normally used in flowmetry (see Solved problem 8.10). If we know the diameter of the vessel, it is possible to compute the flow rate in the vessel.

## SOLVED PROBLEM 8.10

### Doppler flowmetry

The average velocity of blood flow in an artery is $1.7 \cdot 10^{-2}$ m s$^{-1}$. Calculate the frequency change in a Doppler flowmeter with a $10^6$ Hz frequency source (the speed of sound in the blood is 1570 m s$^{-1}$).

### *Solution*

Using the expression (8.46) we have

$$\Delta f = f\frac{2u}{u+v} = \frac{10^6\ \text{Hz} \cdot 2 \cdot 1.7 \cdot 10^{-2}\ \text{m s}^{-1}}{1570\ \text{m s}^{-1} + 1.7 \cdot 10^{-2}\ \text{m s}^{-1}} = 21.66\ \text{Hz}.$$

But how can we distinguish two ultrasounds that are so close in frequency? These are frequencies emitted by the ultrasonic probe of $f_1$ = 1 000 000 Hz and in the case of an approaching flow of $f_2$ = 1 000 021.7 Hz!

It is a question of distinguishing 1 Hz out of a million Hz. Here, the beats from these very similar frequencies ($f_1 \approx f_2$) come to our aid (Paragraph 8.6.2). Indeed, on adding the two ultrasounds, the variation in amplitude of the resulting ultrasound changes with a frequency equal to $(f_2 - f_1)/2$. Therefore, by measuring the changes in the resulting amplitude, it is easy to obtain the change in frequency, however small.

## 8.12.2 Therapy

Ultrasounds can cause (at low intensity) a localized thermal effect (**diathermy**) in the tissues without damaging the cells.

Several ultrasound waves can be focused on the same point, thus creating a so-called shock wave (i.e., a high-intensity sound pulse). Shock waves can be used to shatter kidney stones (**Shock Wave Lithotripsy**, SWL). In this case, a device (called **lithotripter**) focuses ultrasonic rays on the patient's stone, under the guidance of real-time ultrasound scans (**Figure 8.33**). The shock wave pressure must be much greater than the resistance limit of the stone, but at the same time lower than the tolerance limit of biological tissues. The pressure values on the stone are in the order of 50÷100 MPa (about 500÷1000 atm) for 1÷5 μseconds.

The effect is similar to a hammer blow on the stone (as explained in Solved problem 8.11), shattering it to very small fragments, which are then removed by the urinary system.

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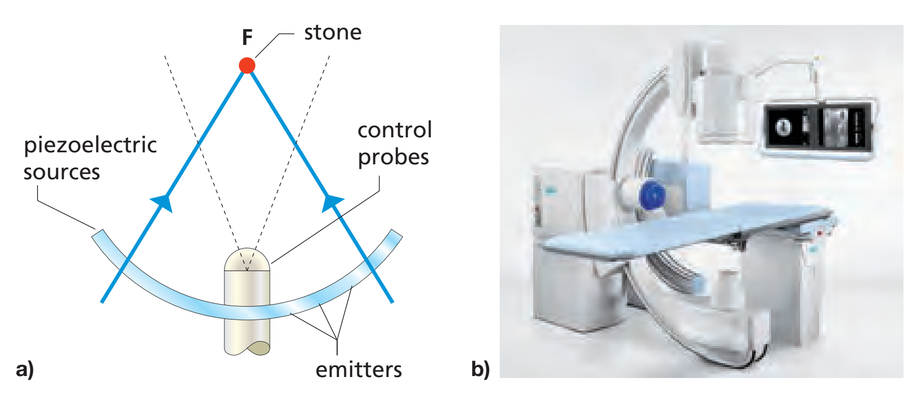


**Figure 8.33** Piezoelectric crystal lithotripter. **a)** Operating diagram in which the parabolic matrix of piezoelectric elements and the ultrasound probe for control imaging are contained in a cushion filled with water in contact with the patient. The stone must be placed in the focus F. **b)** Overall view of the device with the radioscopic monitor and the ultrasound monitor, the probe of which is integrated into the system of piezoelectric emitters shown in **a)**. (*Credit*: b) Siemens Healthineers AG.)

## SOLVED PROBLEM 8.11

### Lithotripter force acting on a kidney stone

A lithotripter exerts a pressure of 60 Mpascal on a spherical kidney stone with a diameter of 7 mm. Compare this force with the force exerted on a body similar to the kidney stone by a hammer with a mass of 2.5 kg falling from a height of one meter.

### *Solution*

The hammer of mass $m$, accelerating under the action of the weight force $F_w$, hits the body at speed $v$. As the body is placed on a rigid, fixed surface, the hammer stops, owing to the instantaneous constraint reaction exerted by the surface, which is equal in modulus, but opposite to the force $F_w$. From Newton's second law, we have, in modulus, $F_w = -mg = -m\Delta v/\Delta t$, where $g$ is the deceleration of the hammer, $\Delta t$ is the time taken by the hammer to decelerate on contact with the body, which we can estimate as 0.005 seconds, and where $\Delta v$ is the change in speed following the impact. The final speed $v_f$ after the impact will be zero, while the initial speed can be calculated by applying the law of conservation of mechanical energy, described in Eq. (1.30): $\frac{1}{2}mv^2 = mgh$, which gives

$$v = \sqrt{2gh} = 4.43\ \mathrm{m\,s^{-1}}.$$

The force $F_w$ of the hammer is therefore

$$F_w = \frac{m\Delta v}{\Delta t} = \frac{2.5\ \mathrm{kg} \times 4.43\ \mathrm{m\,s^{-1}}}{0.005\ \mathrm{s}} = 2215\ \mathrm{N}.$$

The force $F_p$ corresponding to a pressure of 60 MPa on a surface of $S = \pi r^2 =$ $= \pi(0.35\ \mathrm{cm})^2 = 0.38 \cdot 10^{-4}\ \mathrm{m}^2$ is

$$F_p = pS = 60 \cdot 10^6\ \mathrm{Pa} \times 0.38 \cdot 10^{-4}\ \mathrm{m}^2 = 2280\ \mathrm{N}.$$

Therefore comparing $F_w$ and $F_p$, a single acoustic impulse from the lithotripter causes effects similar to those of a 2.5 kg hammer dropped from a height of one meter onto a stone of 7 mm in diameter.

Ultrasounds are used in dentistry to eliminate tartar, in ophthalmology in cataract operations to destroy the lens (the fragments of which are eliminated by aspiration) and in cosmetic surgery to reduce adiposity. For the treatment of tumors, a technique called **HIFU** (**High-Intensity Focused Ultrasound**) has been developed to precisely destroy tissue by means of focused ultrasound absorption.

## PROBLEMS

**8.1** ■ A 600 Hz tuning fork is at rest and an identical one is moving toward it. When excited, they produce a beat frequency of 2 Hz. Calculate the velocity of the tuning fork in motion (velocity of sound $v$ = 343 m/s).

[$v$ = 2.29 m $s^{-1}$]

**8.2** ■ An organ pipe, 1.2 m long and open at both ends, is placed next to another pipe, which is closed at one end and 61 cm long. Given that each pipe resonates with its own fundamental harmonic, calculate the frequency of the beats (velocity of sound $v$ = 343 m/s).

[$f_{beats}$ = 1.171 Hz]

**8.3** ■ An observer moves in the direction of a stationary source and perceives a frequency variation of 7% due to Doppler effect. Determine the velocity of the observer (velocity of sound $v$ = 343 m/s).

[$v_o$ = 24.01 m $s^{-1}$]

**8.4** ■ Calculate the intensity of a sound wave in air in STP conditions (speed of sound 340 m/s) if its frequency is 1300 Hz and its amplitude is 13 μm. The air density is 1.29 kg/$m^3$.

[$I$ = 2.47 W $m^{-2}$]

**8.5** ■ Two sounds have intensity of 25 and 670 microwatts/$cm^2$ respectively. How many decibel is the most intense sound greater than the other?

[$\sigma$ = 14.28 dB]

**8.6** ■ In a room there are two loudspeakers. At a given point of the room a person receives the sounds from both sources and their intensity levels are respectively $IL_1$ = 87 dB and $IL_2$ = 87 dB (same value for the two sources). Which is the total Intensity Level heard by the listener?

[IL = 90 dB]

**8.7** ■ Calculate the pressure amplitude of a thunder whose intensity is 0.5 W/$m^2$ (air density $d$ = 1.29 kg/$m^3$ and speed of sound = 343 m/s).

[$\Delta p$ = 21.035 Pa]

**8.8** ■ An average frequency shift of 45 Hz is detected in a Doppler flow meter working with a 1.8 MHz frequency source. Calculate the average velocity of blood flow in the vessel (the speed of sound in the blood is 1570 m/s).

[$v$ = 1.96 cm/s]

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