

How to write numbers

- Numbers can be written as symbols (10) or words (ten).
- In formal writing use symbols for large amounts and words for everything else.
- A decimal point is written as a “dot”, not a comma.
example 6.5 six point five
- A comma can be used to separate hundreds from thousands, from millions, ...
example 3,498,570

How to read numbers

- Numbers after the decimal dot, are read separately
- The “zero” before a dot can be read as “nought” or not be read at all.
example 0.25 (nought/zero) point two five (**not** twentyfive)
- When reading a big number, do not use plural for “million”, “thousand” and “hundred”
example 6,200 six thousand two hundred (**not** thousands, **not** hundreds)
- One difference between British English (BrE) and American English (AmE) is the use of “and” when reading big numbers.
example 5,370 five thousand three hundred and seventy (in BrE, in AmE there is no “and”)

Scientific notation

- Numbers in scientific notation are written as:
 $a \times 10^b$ (“a times ten to the power of b”)
 The exponent b is an integer, and the coefficient a is a real number between 1 and 10 called the *significant* or *mantissa*.

Order of magnitude

- The rounding off of a number to the nearest power of 10.

square root

left (round) bracket

cubed (to the third)

curly bracket

point two five

square bracket

three fourths

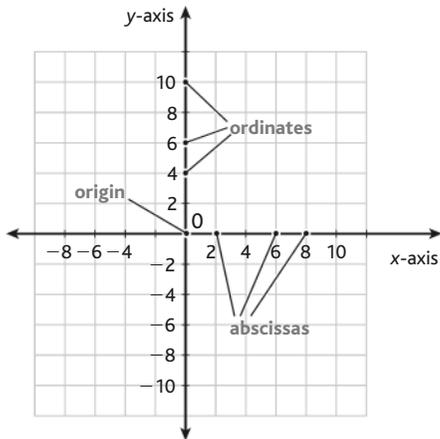
one half

$$\sqrt{\left\{ (0.25 \cdot 12) - \left[1 - \left(\frac{1}{3} + \frac{1}{6} \right) \right] \right\}^3 - \left(\frac{3}{4} + \frac{1}{2} - 3.5 \right)^2} = \frac{13}{4}$$

In symbols	In words	Examples
+	plus, add	$a + b$ a plus b
-	minus, take away, subtract	$a - b$ a minus b
±	plus or minus	
× · (dot product)	times, multiplied by	$a \times b$ ab , a times b $a \cdot b$ ab , a times b
÷ ⋮ ⋮ (vinculum or fraction bar)	divided by	$\frac{a}{b}$ a over b , a divided by b $\frac{a}{b}$ in fractions, a is called the <i>numerator</i> and b the <i>denominator</i> how to read fractions $\frac{1}{2}$ $\frac{5}{2}$ $\frac{2}{3}$ $\frac{7}{10}$ $\frac{\pi}{4}$... one half, five halves, two thirds, seven tenths, pi over four, ...
=	is equal, equals, is	$a = b$ a equals b or a is equal to b $1 + 2 = 3$ one plus two is (equals) three
≈	is approximately equal to	
≠	is not equal to	$a \neq b$ a is different from b , a is not equal to b
< > ≪ ≫ ≥ ≤	inequality signs	$a < b$ a is (strictly) less than b $a > b$ a is (strictly) greater than b $a \ll b$ a is much less than b $a \gg b$ a is much greater than b $a \geq b$ a is greater than or equal to b $a \leq b$ a is less than or equal to b
%	percent	5% five percent
α Δ ∞ a ₁ a' ā	is proportional to delta, change in infinity a sub 1 a prime vector \bar{a}	$a \propto b$ a is proportional to b Δa the change in a is given by the final value minus the initial value in a

In symbols	Name	In words
a^b In particular: x^2 x^3 \sqrt{x} $\sqrt[3]{x}$	exponential	<p><i>a raised to the power of b</i> <i>a to the b</i> <i>a to the b-th (power) [if b is a positive integer]</i></p> <p>examples 2^6 two to the power of six 3^{-4} three to the power of minus four</p> <ul style="list-style-type: none"> <i>x squared</i> $5^2 = 25$ five squared is twentyfive <i>x cubed</i> $5^3 = 125$ five cubed is one hundred and twentyfive (BrE) <i>square root of x</i> $\sqrt{36} = 6$ square root of thirtysix equals (is) six <i>the cube root of x</i> $\sqrt[3]{216} = 6$ cubic root of two hundred and sixteen is six (BrE)
ln... $\log_a...$	natural logarithm of logarithm to base a	when the base of the logarithm "log" is not specified, ten (10) is implied $\log x$ logarithm to base ten of x
sin cos tan cot	sine cosine tangent cotangent	$\sin \frac{\pi}{6} = \frac{1}{2}$ the sine of pi over six is one half $\tan \frac{\pi}{4} = 1$ the tangent of pi over four is one

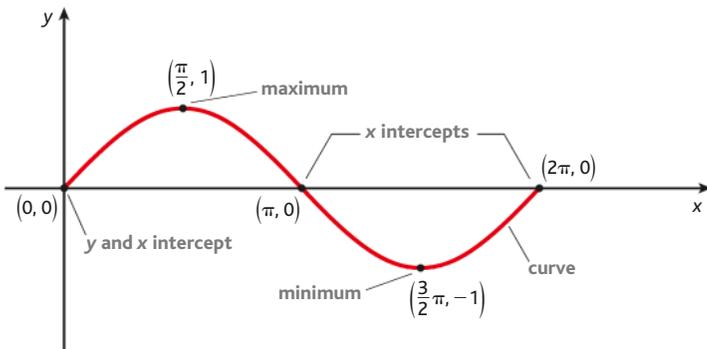
Cartesian plane



In mathematics, the graph of a function f is the collection of all ordered pairs $(x, f(x))$.

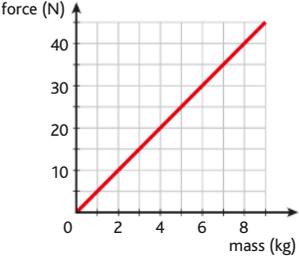
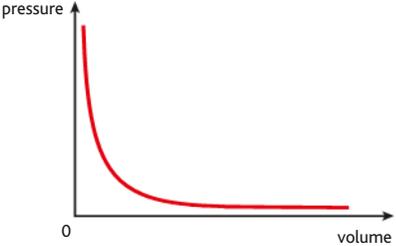
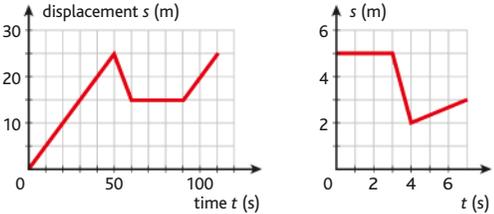
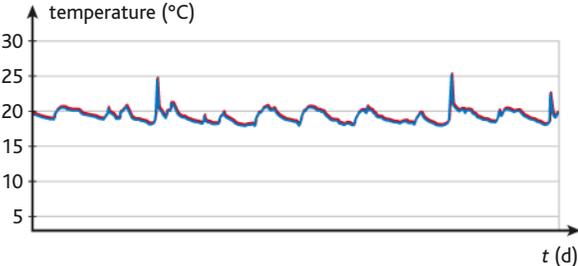
Graphing on a cartesian plane is sometimes referred to as to *plot* or *draw* a curve.

Main features of the graph of a function

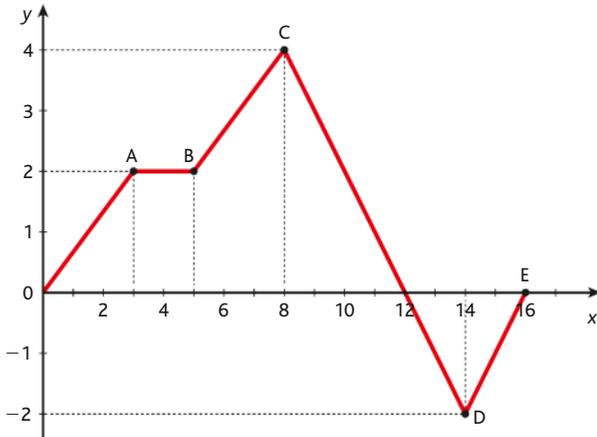


- **Domain:** the set of x -coordinates corresponding to the points on a graph
In the example, the *domain* is $[0; 2\pi]$ (zero; two pi).
- **Range:** the set of y -coordinates corresponding to the points on a graph.
In the example above, the *range* is $[-1; 1]$ (minus one; one).
- **x -intercept:** the point where the graph intersects the x -axis.
In the example, there are three x -intercepts, $(0, 0)$, $(\pi, 0)$ and $(2\pi, 0)$.
- **y -intercept:** the point where the graph intersects the y -axis.
In the example, the only y -intercept is $(0, 0)$.
- **Maximum/minimum:** the greatest/lowest value assumed by a function over a given interval.
In the example above, the function attains the maximum value 1 at $x = \frac{\pi}{2}$
and the minimum value -1 at $x = \frac{3}{2}\pi$.

Describe the trend of a graph

Graph	Description
	<p><i>Stays the same/is flat/remains unchanged</i> The current remains unchanged over time.</p>
	<p><i>Rises/increases/grows</i> The force increases as the mass increases. The force increases with the mass. The force is directly proportional to the mass.</p>
	<p><i>Falls/drops/declines/decreases</i> The pressure decreases as the volume increases. The pressure is inversely proportional to the volume.</p>
	<p><i>Peaks/reaches a peak</i> <i>Hits a low</i></p> <p>In the graph on the left, displacement reaches a peak when time is 50 s. In the graph on the right, displacement hits a low when time is 4 s.</p>
	<p><i>Fluctuates</i> In the graph, temperature fluctuates around a mean value 20 °C over time.</p>

Read a graph



Increasing/decreasing

y increases proportionally to x when x is between 0 and 3;

y remains constant when x is between 3 and 5;

y increases again when x is between 5 and 8;

it decreases and increases again...

Peaks and lows

for the graph, y reaches a peak in $(8, 4)$;

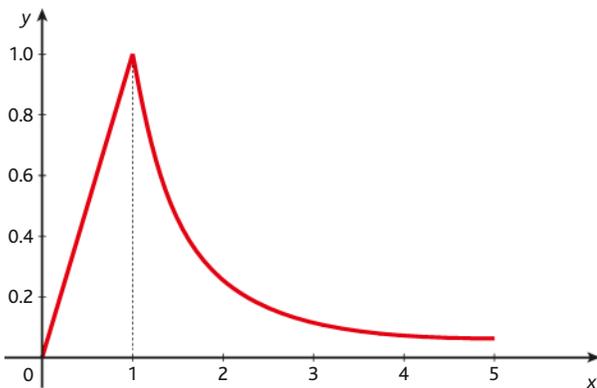
for the graph, y hits a low in $(14, -2)$.

Intersections with the axes

the graph intersects the x -axis in $(0, 0)$, $(12, 0)$ and $(16, 0)$.

Domain: $[0; 16]$

Range: $[-2; 4]$



Increasing/decreasing

y increases proportionally to x when x is between 0 and 1, then it decreases.

Peaks and lows

for the graph, y reaches a peak when x is 1.

Intersections with the axes

the graph passes through the origin, so the x and y -intercept is $(0, 0)$.

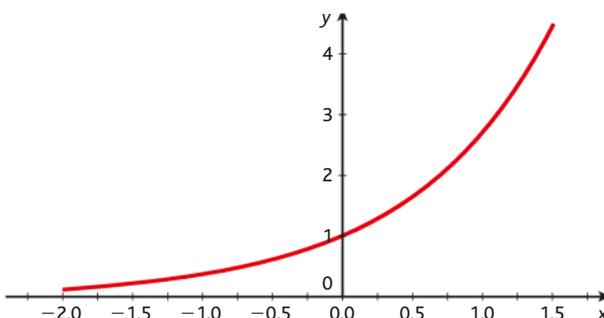
Domain: $[0; +\infty[$

Range: $[0; 1]$

Behaviour at infinity

when x approaches to infinity, y approaches 0.
($y = 0$ is an *asymptote* of the curve)

Example: the exponential function $y = a^x$



The domain of function f is the set of all real numbers. The range of f is the interval $[0, +\infty[$.

The graph approaches the x -axis as an asymptote.

(The graph of f has a horizontal asymptote given by $y = 0$.)

Function f has a y intercept at $[0; 1]$.

f is an increasing function if a is greater than 1 and a decreasing function if a is smaller than 1.



Formulae

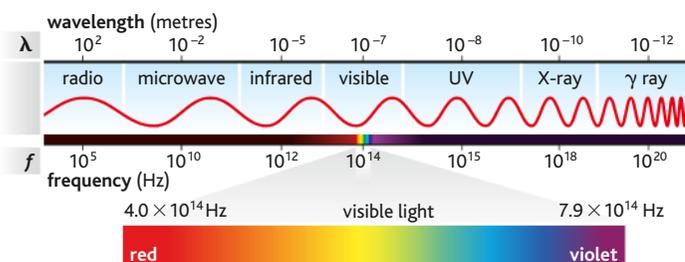
Subject	In symbols	In words
Newton's second law of motion	$\vec{F} = m\vec{a}$	Force equals mass multiplied by acceleration.
Linear momentum	$\vec{p} = m\vec{v}$	The linear momentum of a body equals the product of the mass of a body and its velocity.
Coulomb's law	$F = k_0 \frac{q_1 q_2}{r^2}$	The electrostatic force acting simultaneously between two point charges is equal to the product of the proportionality constant k_0 , the charges q_1 and q_2 , and the reciprocal of the square of the separation distance r of the point charges.
Electric potential energy	$U = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r}$	The electric or electrostatic potential energy of charge q_1 in the potential of charge q_2 is equal to the product of the reciprocal of four pi multiplied by the permittivity ϵ of the medium, the charges q_1 and q_2 , and the reciprocal of the separation distance r of the point charges.
Energy of a photon	$E = hf$	The energy of a photon is equal to the product of the Planck's constant h and the frequency f of its associated electromagnetic wave.
Momentum of a photon	$p = \frac{h}{\lambda} = \frac{E}{c}$	The momentum of a photon is equal to the ratio of the Planck's constant h to the wavelength λ of its associated electromagnetic wave, or the ratio of its energy E to the speed of light.
Heisenberg uncertainty principle	$(\Delta x)(\Delta p_x) = \frac{h}{4\pi}$	In describing an elementary particle, the uncertainty in the position (Δx) multiplied by the uncertainty in its momentum (Δp_x) is equal to Planck's constant divided by four pi.
Rest energy	$E_0 = m_0 c^2$	The total energy of a body at rest is equal to the product of its rest mass m_0 (also called invariant mass) and the square of the speed of mass.
Radioactive decay	$N = N_0 e^{-\lambda t}$	For a given sample of a specific radioisotope, the number of atoms present after a period of time t is equal to the initial number of atoms N_0 (at time $t = 0$) multiplied by the exponential function of $2\lambda t$, where λ is the decay constant for the radioisotope and t is the elapsed time.
Half-life	$T_{1/2} = \frac{\ln 2}{\lambda}$	The half-life of a substance undergoing decay, i.e. the period of time in which the expected number of entities that have decayed is equal to half the original number, is equal to the natural logarithm of two divided by the decay constant λ for the substance; λ being a positive number.

Concepts and laws

Law	Definition
Conservation of energy	Energy can neither be created nor destroyed but it can be changed from one form to another.
Quantisation of electric charge	Electric charge is quantised in units of electron charge. The total charge carried by any object is a whole multiple of the electron charge.
Conservation of electric charge	Whenever a certain amount of charge is produced on one object, an equal amount of the opposite type of charge is produced on another object. Electric charge can be neither created nor destroyed.
Heat engine	A thermodynamic system, operating in a cycle, that converts heat or thermal energy into mechanical work and operates between two heat reservoirs at a fixed temperature. The system absorbs heat from the hot reservoir Q_H <ul style="list-style-type: none"> • a fraction of the heat is converted into work W; • an amount of heat is transferred to the cooler thermal reservoir Q_C.
Efficiency	The efficiency of a heat engine is the ratio of the work done by the engine to the heat energy input into the engine. Heat engines obey the principle of conservation of energy: $W = Q_H - Q_C$.

The electromagnetic spectrum

The range of all possible frequencies or wavelengths of electromagnetic radiation.
The visible spectrum: electromagnetic waves with frequencies between about 4×10^{14} Hz and 7.9×10^{14} Hz.



The profile of electromagnetic waves

Electromagnetic waves are transverse waves for which the oscillating electric and magnetic fields are both perpendicular to the propagation direction.
For each point in space, the amplitude of an electromagnetic wave oscillates as a function of time at the same frequency f as the source.
The period T and the wavelength λ in vacuum can be calculated from the propagation speed c .

“Whenever we proceed from the known into the unknown we may hope to understand, but we may have to learn at the same time a new meaning of the word “understanding”.

(Werner Heisenberg, 1932 Nobel Prize in Physics)

“A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.

(Max Planck, 1918 Nobel Prize in Physics)

“We always have had ... a great deal of difficulty in understanding the worldview that quantum mechanics represents. At least I do, because I'm an old enough man that I haven't got to the point that this stuff is obvious to me. Okay, I still get nervous with it. And therefore, some of the younger students ... you know how it always is, every new idea, it takes a generation or two until it becomes obvious that there's no real problem.

(Richard Feynman, 1965 Nobel Prize in Physics)