LEARNING OBJECT 0 The Basic Moves
Pre-requisite Knowledge

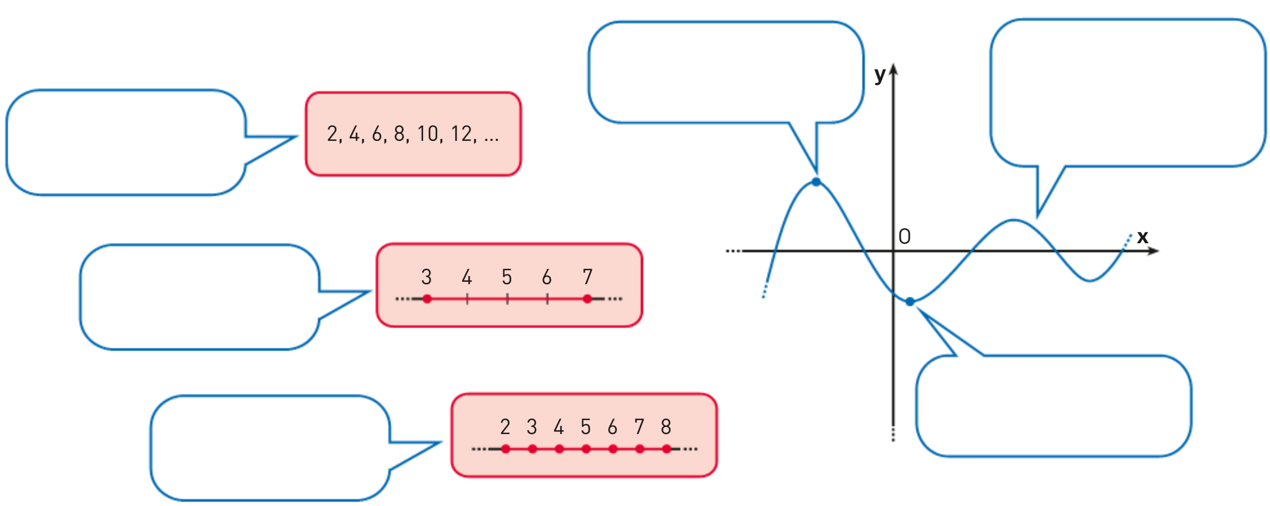
Activity 1 10 min **Complete the following sentences with the options in the table below.**

Hands-on glossary
- **set**: insieme
- **partition**: partizione, suddivisione
- **sequence**: successione
- **domain**: dominio
- **to lift**: sollevare
- **length**: lunghezza
- **range**: immagine

a. An interval is the **set** of all ....
b. A uniform **partition** of an interval is a ....
....
c. A sequence is a **function** whose domain is the set of natural numbers and
....
d. $\int 15t^4\,dt =$ ....
e. A graph of a continuous function on an interval can ....
f. Functions can have “hills”: places where ....
g. Functions can have “valleys”: places where ....
h. $\int (\sqrt{2t+1} + 6)\,dt =$ ....

…they reach a local minimum value.
…be drawn without **lifting** the pencil from the paper.
…numbers between two given numbers.
…$3t^5 + c$
…set of points that divide the interval into subintervals of the same **length**.
…they reach a local maximum value.
…whose **range** is a subset of the real numbers.
…$\frac{1}{3}\sqrt{(2t+1)^3} + 6t + c$

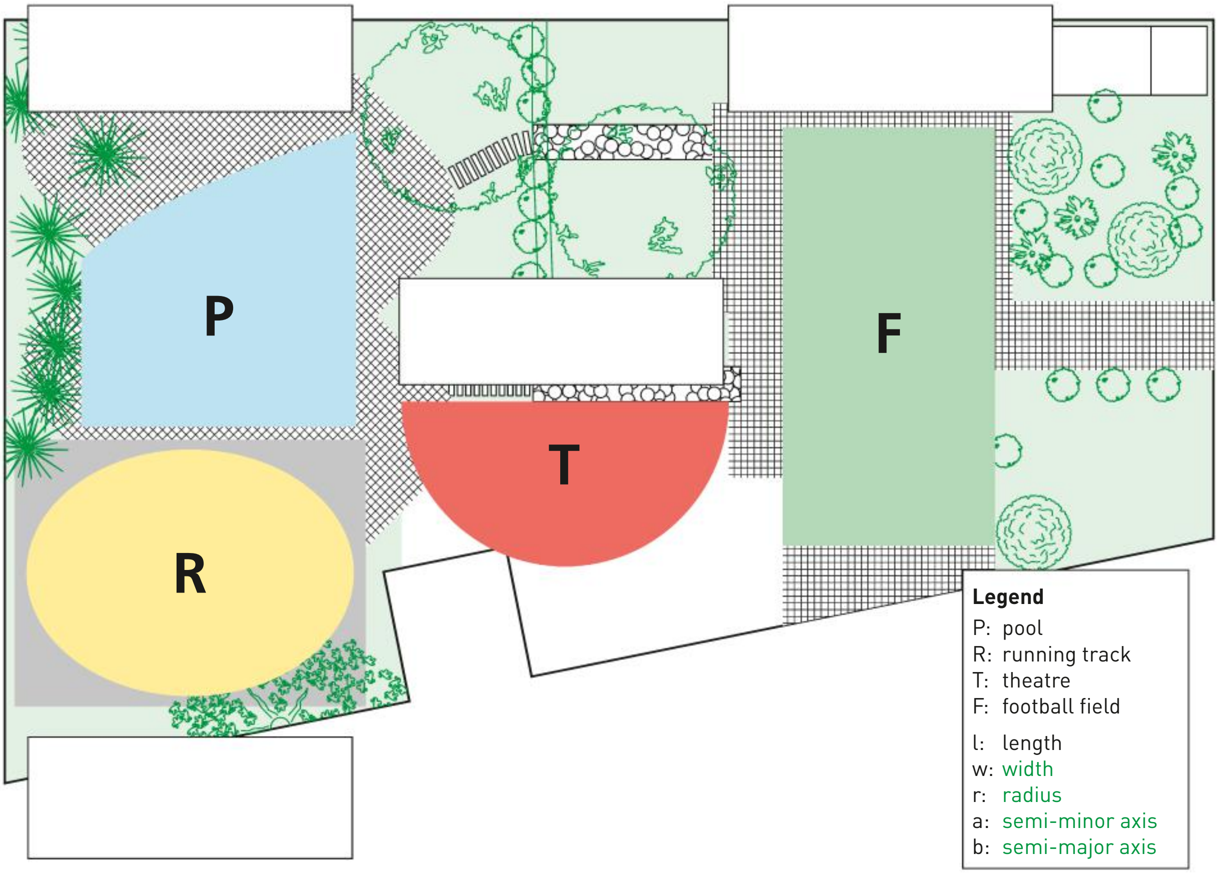
Activity 2 5 min **Label the pictures using the underlined terms from Activity 1.**



1 LEARNING OBJECT Rectangles Take the Field

Inscribed Rectangles and Circumscribed Rectangles

Mr. Riemann, the Director of Integration High School, is considering the cost of building some facilities at his school. Below is a potential plan for the new facilities. To calculate the cost of each option he needs to calculate the surface area to be cemented.



Activity 3 5 min **Use these formulas for calculating the areas or the statement to fill in the boxes in the figure above. Note the legend below the figure, on the right.**

$A = lw$ • $A = \frac{1}{2}\pi r^2$ • $A = \pi ab$ • A new kind of formula is needed

HANDS-ON GLOSSARY

facility: struttura

surface area: superficie

width: larghezza

to fill in: compilare, riempire

radius: raggio

semi-minor axis: semiasse minore

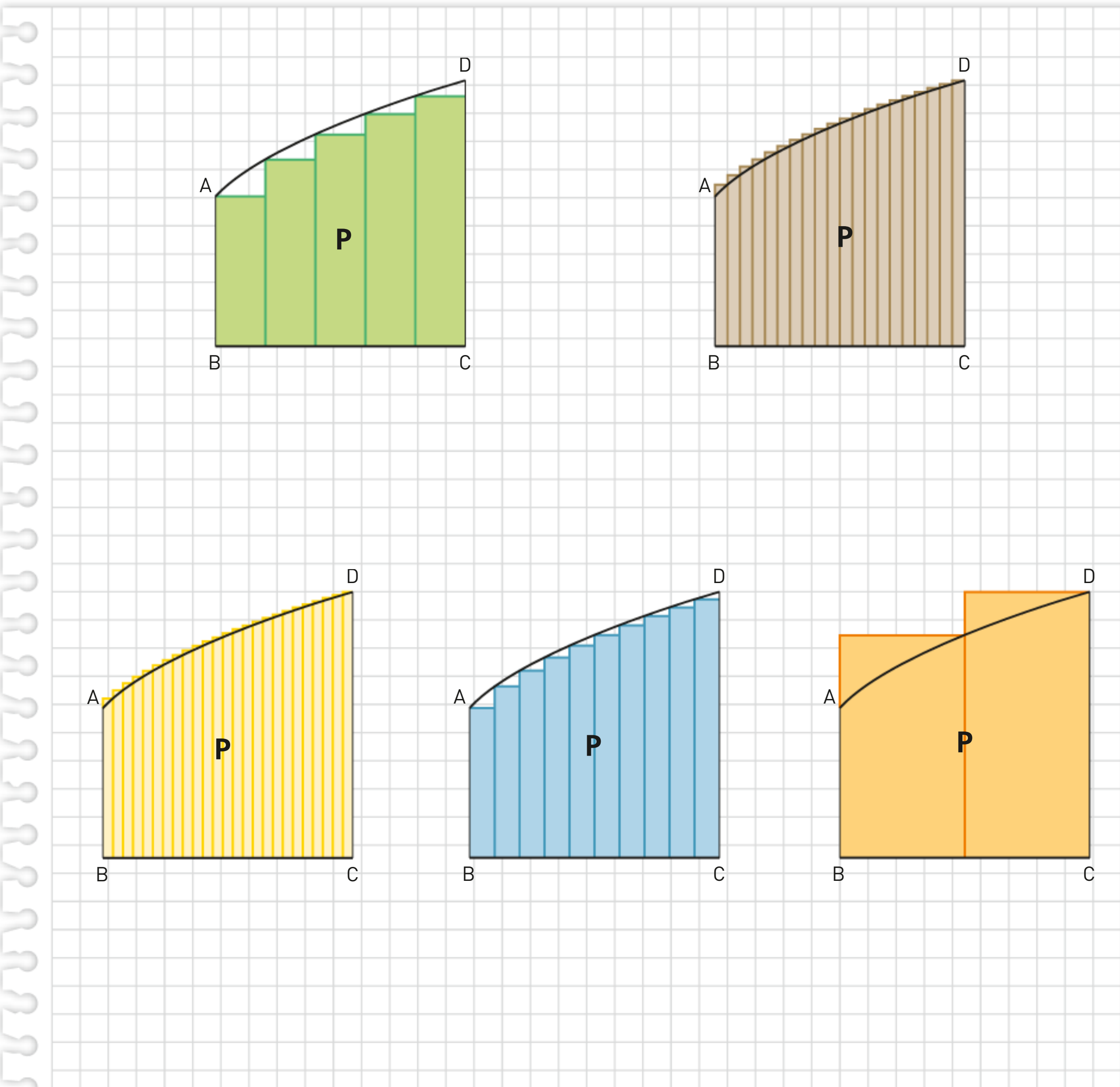
semi-major axis: semiasse maggiore

Activity 4 5 min **P is the region where a pool will be built. According to Mr. Riemann, rectangles are the best way to estimate the area of P, which has a non-standard shape. Complete Mr. Riemann's notes by writing the following sentences under the corresponding picture.**

HANDS-ON GLOSSARY

estimate: stima
circumscribed: circoscritto
inscribed: inscritto

This estimate of the area of P uses 2 circumscribed rectangles.
This estimate of the area of P uses 5 inscribed rectangles.
This estimate of the area of P uses 10 inscribed rectangles.
This estimate of the area of P uses a very large number of circumscribed rectangles.
This estimate of the area of P uses approximately 20 circumscribed rectangles.



Activity 5 5 min **Mr. Riemann made some interesting observations. Choose the two sentences that summarize Mr. Riemann's conclusions and write them in the thought balloons.**

a. The fewer the rectangles forming the union of rectangles, the more precise the estimate of the area of P.

b. The more rectangles forming the union of rectangles, the more precise the estimate of the area of P.

c. The narrower the rectangles forming the union of rectangles, the more precise the estimate of the area of P.

d. The wider the rectangles forming the union of rectangles, the more precise the estimate of the area of P.

Hands-on glossary

fewer: meno

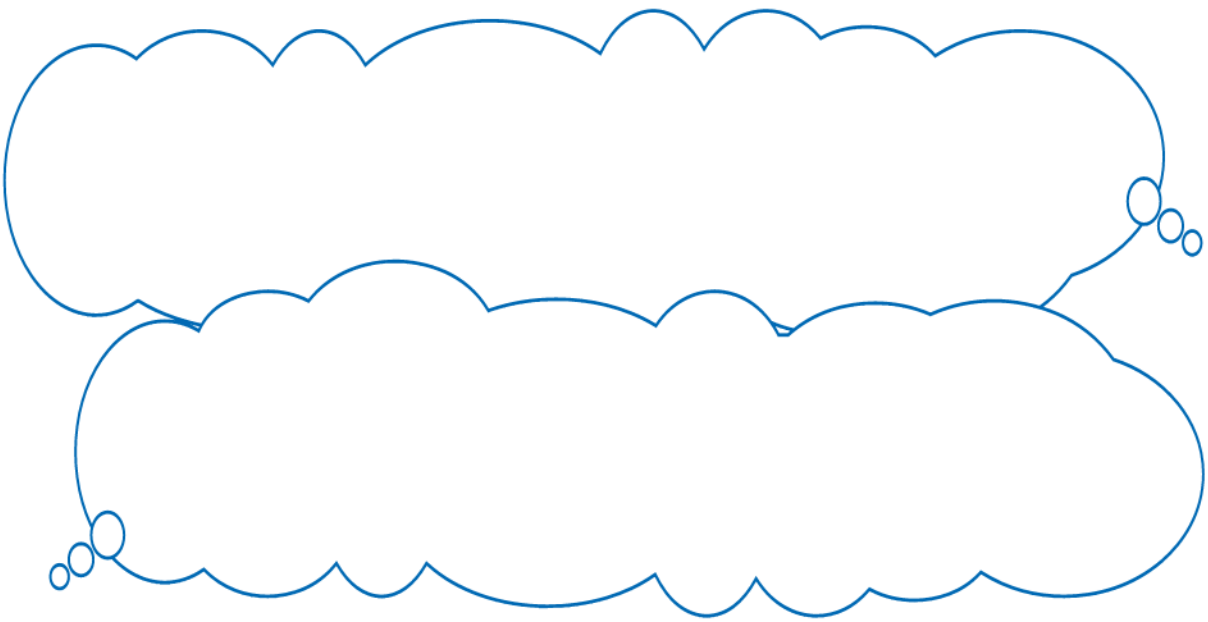
union of rectangles: plurirettangolo

more: più

narrow: stretto

wide: largo

to approach: avvicinarsi



Activity 6 8 min **Use the words below to help Mr. Riemann complete his notes about the area of P. We will then listen to the recording to check activities 5 and 6.**

Listening

area • circumscribed • curve • greater • more • result • zero

To calculate the .... of the region under the .... , we could use inscribed or circumscribed union of rectangles.

The estimated .... is .... accurate if we use unions of rectangles made up of a .... number of rectangles.

Moreover, as the bases of the rectangles approach .... in width, the area covered by both the inscribed and the .... union of rectangles becomes almost the same.

This is the area of P.

LEARNING OBJECT 2

Lower and Upper Face Off

Introduction to Lower Sums and Upper Sums

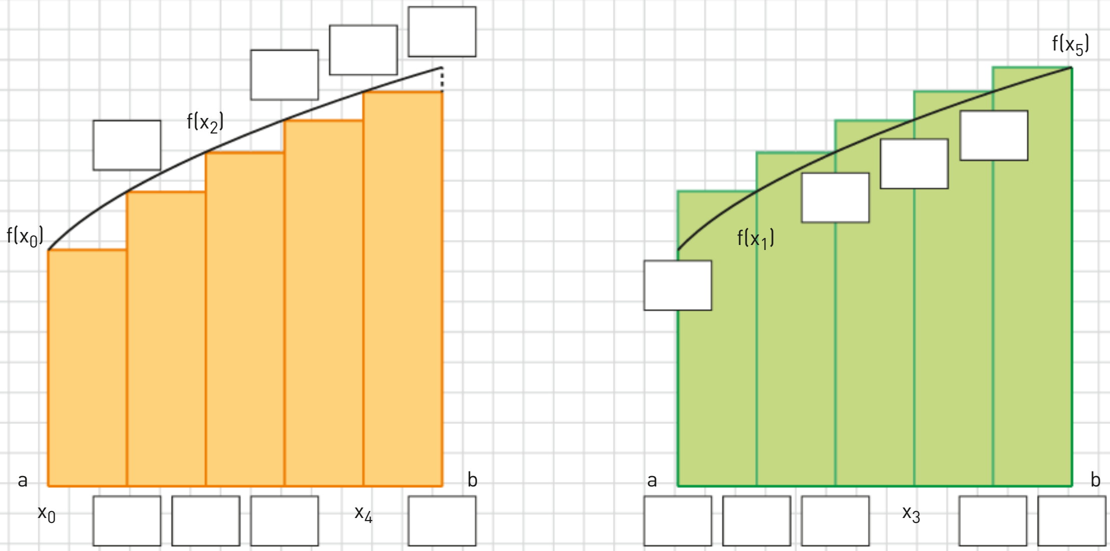
Mr. Riemann has drawn two **sketches** of the pool, **both** containing five subintervals with the same uniform partition over interval $[a; b]$. As you know, the black line represents the graph of a *continuous* and *positive* real function.

HANDS-ON GLOSSARY

sketch: schizzo

both: entrambi

Activity 7 10 min **Using your prior knowledge, fill in the missing labels in both graphs below.**



Activity 8

10 min

Write these sentences under the corresponding graph on the previous page. Some can be used twice.

a. The width of each subinterval can be indicated by: $\Delta x = \dfrac{b-a}{5}$.

b. In the inscribed union of rectangles, $f(x_{i-1})$ represents the **minimum** (m_i) of the function within each subinterval.

c. In the circumscribed union of rectangles, $f(x_i)$ represents the **maximum** (M_i) of the function within each subinterval.

d. Each m_i represents the height of each inscribed rectangle.

e. Each M_i represents the height of each circumscribed rectangle.

f. The area of this union of rectangles is $s_5 = \sum_{i=1}^{5} m_i \cdot \Delta x$.

g. The area of this union of rectangles is $S_5 = \sum_{i=1}^{5} M_i \cdot \Delta x$.

h. The area of this union of rectangles is the lower sum.

i. The area of this union of rectangles is the upper sum.

Hands-on glossary

- **height**: altezza
- **lower sum**: somma inferiore
- **upper sum**: somma superiore
- **to lie**: trovarsi
- **less**: minore, più piccolo
- **great**: grande

Activity 9

5 min

Mr. Riemann has reached two more correct conclusions: which are they?

The area under the curve $f(x)$, that is, the area of P, lies between s_5 and S_5.

A

The area under the curve $f(x)$, that is, the area of P, is greater than S_5.

B

The area under the curve $f(x)$, that is, the area of P, is S_5.

C

The area under the curve $f(x)$, that is, the area of P, is less than s_5.

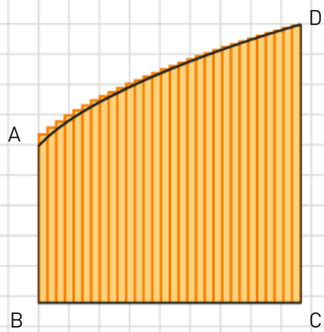
D

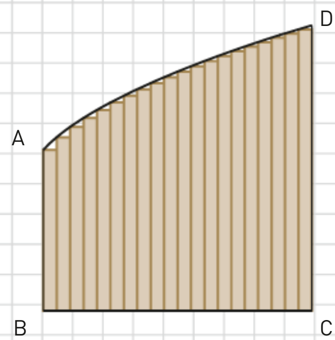
The area under the curve $f(x)$, that is, the area of P, cannot be less than s_5.

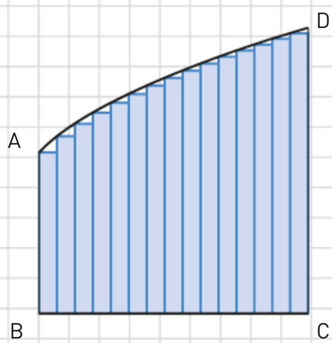
E

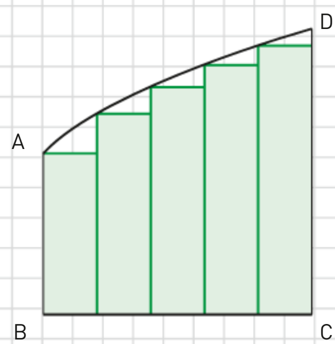
Activity 10 10 min **In this table, there are nine formulas for calculating the lower and the upper sums of a variety of partitions of interval $[a; b]$.**
Below, there are six graphs showing a variety of partitions of interval $[a; b]$. Write the correct formula under the corresponding graph.

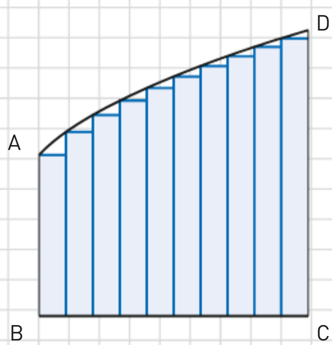
$S_{10} = \sum_{i=1}^{10} M_i \cdot \Delta x$	$S_{20} = \sum_{i=1}^{20} M_i \cdot \Delta x$	$S_5 = \sum_{i=1}^{5} M_i \cdot \Delta x$
$s_5 = \sum_{i=1}^{5} m_i \cdot \Delta x$	$s_{20} = \sum_{i=1}^{20} m_i \cdot \Delta x$	$s_{10} = \sum_{i=1}^{10} m_i \cdot \Delta x$
$s_{15} = \sum_{i=1}^{15} m_i \cdot \Delta x$	$S_{30} = \sum_{i=1}^{30} M_i \cdot \Delta x$	$S_{15} = \sum_{i=1}^{15} M_i \cdot \Delta x$

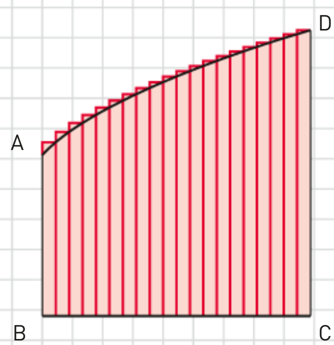












3 Lower and Upper Team Up
Lower Sums and Upper Sums

Activity 11 10 min **Are the following inequalities about the area of P true (T) or false (F)?**

a.	$s_5 < \text{Area}(P) < S_5$	T F
b.	$s_5 < s_{10} < \text{Area}(P) < S_{10} < S_5$	T F
c.	$S_{10} < \text{Area}(P) < s_{10}$	T F
d.	$s_{10} < s_5 < \text{Area}(P) < S_5 < S_{10}$	T F
e.	$s_{10} < s_{15} < \text{Area}(P) < S_{15} < S_{10}$	T F
f.	$S_{20} < s_5 < \text{Area}(P) < S_5 < s_{20}$	T F
g.	$S_{10} < S_{15} < \text{Area}(P) < s_{15} < s_{10}$	T F
h.	$s_{20} < s_{30} < s_{n\to+\infty} < \text{Area}(P) < S_{n\to+\infty} < S_{30} < S_{20}$	T F
i.	$s_{30} < s_{n\to+\infty} < s_5 < \text{Area}(P) < S_5 < S_{n\to+\infty} < S_{30}$	T F
j.	$s_{n\to+\infty} < s_{30} < \text{Area}(P) < S_{30} < S_{n\to+\infty}$	T F

Activity 12 8 min **Use the following words to complete the summary of what we have observed.**

area • greater • increases • overestimate • same • sequence • underestimate

Hands-on glossary

inequality: disuguaglianza
to overestimate: sovrastimare
to underestimate: sottostimare
increasing: crescente
decreasing: decrescente
i.e.: cioè

Using **circumscribed** rectangles, and thus the upper sums (S_n), will always lead to an .... of the area of P, whereas using **inscribed** rectangles, and thus the lower sums (s_n), will always lead to an .... of the area of P.

This imprecision is .... when using 10 inscribed or circumscribed rectangles than when using 30. The imprecision decreases when the number of rectangles ....

The lower sums s_5, s_{10}, s_{15}, s_{20}, s_{30}, …, s_n form an increasing .... and the upper sums S_5, S_{10}, S_{15}, S_{20}, S_{30}, …, S_n form a decreasing sequence.

Both of these sequences approach the .... value, i.e. the .... under the curve.

Activity 13 10 min **Let's check our progress. Below are some concepts and their definitions which should be familiar now. How well do you understand them?**

Concept	Definition	Very well	Not really
Inscribed rectangle	A rectangle whose base is the width of the subinterval and whose height is the minimum of the function in the same subinterval.		
Circumscribed rectangle	A rectangle whose base is the width of the subinterval and whose height is the maximum of the function in the same subinterval.		
Inscribed union of rectangles	A set of inscribed rectangles. The number of rectangles can be any natural number.		
Circumscribed union of rectangles	A set of circumscribed rectangles. The number of rectangles can be any natural number.		
Lower sum	The area of the inscribed union of rectangles.		
Upper sum	The area of the circumscribed union of rectangles.		

Activity 14 10 min **Complete the text with the following list of words. Then listen to the recording to check your answers.** Listening

area • bound • circumscribed • greater • inscribed • lower • number • union of rectangles • under • upper

Mr. Riemann tried to calculate the area of *P* by **covering** the area under the curve with rectangles. He used two kinds of rectangles: .... rectangles and circumscribed rectangles.
With the inscribed rectangles he built the inscribed .... and with the circumscribed ones he built the .... union of rectangles.

The sum of the areas of the circumscribed rectangles is called the upper sum, and the sum of the areas of the inscribed rectangles is called the .... sum. Obviously, the area of *P* under the curve is greater than the lower sum but less than the .... sum.

The upper sum is the **upper bound**, meaning that the area .... the curve must be less than this upper bound. By contrast, the lower sum is the **lower bound** for the area under the curve, which means that the area under the curve must be .... than the lower sum.
With five rectangles the upper .... and the lower bound are very different. However, with ten rectangles, the upper bound approaches the lower one, while the area of *P* is always between them.

We're definitely making progress, but how many rectangles should we use to get an acceptable value for the .... ? We could go on, using more and more rectangles, but obviously this is not very efficient, and the effort increases with a greater .... of rectangles.
Riemann thus proposed that it would make more sense to find a **general expression** which can be used to calculate the area using any number of rectangles, and then work with that.

Hands-on glossary

to cover: coprire
upper bound: estremo superiore
lower bound: estremo inferiore